NCERT Solutions Class 12 Maths Chapter 1 Exercise 1.2

Question 1:

Show that the function $f : R_* \to R_*$ defined by $(x) = \frac{1}{x}$ is one –one and onto, where R_* is the set of all non –zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?

Solution:

$f : R_* \to R_*$ is by $f(x) = \frac{1}{x}$

For one-one:

$x, y \in R_*$ such that $f(x) = f(y)$

$\Rightarrow \frac{1}{x} = \frac{1}{y}$

$\Rightarrow x = y$

$\therefore f$ is one-one.

For onto:

For $y \in R$, there exists $x = \frac{1}{y} \in R_* \left[\text{as } y \notin 0\right]$ such that

$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$

$\therefore f$ is onto.

Given function f is one-one and onto.

Consider function $g : N \to R$, defined by $g(x) = \frac{1}{x}$

We have, $g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$

$\therefore g$ is one-one.

g is not onto as for $1.2 \in R$, there exist any x in N such that $g(x) = \frac{1}{1.2}$

Function g is one-one but not onto.

Question 2:

Check the injectivity and surjectivity of the following functions:

i. $f : N \to N$ given by $f(x) = x^2$

ii. $f : Z \to Z$ given by $f(x) = x^2$

iii. $f : R \to R$ given by $f(x) = x^2$

iv. $f : N \to N$ given by $f(x) = x^3$

v. $f : Z \to Z$ given by $f(x) = x^3$

Solution:

i. For $f : N \to N$ given by $f(x) = x^2$

$x, y \in N$

$f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$

$\therefore f$ is injective.

$2 \in N$. But, there does not exist any x in N such that $f(x) = x^2 = 2$

$\therefore f$ is not surjective

Function f is injective but not surjective.

ii. $f : Z \to Z$ given by $f(x) = x^2$

$f(-1) = f(1) = 1$ but $-1 \neq 1$

$\therefore f$ is not injective.

$-2 \in Z$ But, there does not exist any $x \in Z$ such that $f(x) = -2 \Rightarrow x^2 = -2$

$\therefore f$ is not surjective.

Function f is neither injective nor surjective.

iii. $f : R \to R$ given by $f(x) = x^2$

$f(-1) = f(1) = 1$ but $-1 \neq 1$

$\therefore f$ is not injective.

$-2 \in Z$ But, there does not exist any $x \in Z$ such that $f(x) = -2 \Rightarrow x^2 = -2$

$\therefore f$ is not surjective.

Function f is neither injective nor surjective.

iv. $f : N \to N$ given by $f(x) = x^3$

$x, y \in N$

$f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$

$\therefore f$ is injective.

$2 \in N$. But, there does not exist any x in N such that $f(x) = x^3 = 2$

$\therefore f$ is not surjective

Function f is injective but not surjective.

v. $f : Z \to Z$ given by $f(x) = x^3$

$x, y \in Z$

$f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$

$\therefore f$ is injective.

$2 \in Z$. But, there does not exist any x in Z such that $f(x) = x^3 = 2$

$\therefore f$ is not surjective.

Function f is injective but not surjective.

Question 3:

Prove that the greatest integer function $f : R \to R$ given by $f(x) = [x]$ is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Solution:

$f : R \to R$ given by $f(x) = [x]$

$f(1.2) = [1.2] = 1, f(1.9) = [1.9] = 1$

$\therefore f(1.2) = f(1.9)$, but $1.2 \neq 1.9$

$\therefore f$ is not one-one.

Consider $0.7 \in R$

$f(x) = [x]$ is an integer. There does not exist any element $x \in R$ such that $f(x) = 0.7$

$\therefore f$ is not onto.

The greatest integer function is neither one-one nor onto.

Question 4:

Show that the modulus function $f : R \to R$ given by $f(x) = |x|$ is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Solution:

$f : R \to R$ is $f(x) = |x| = \begin{Bmatrix} \text{x, if x} \geq 0 \\ -x, \text{if x} < 0 \end{Bmatrix}$

$f(-1) = |-1| = 1$ and $f(1) = |1| = 1$

$\therefore f(-1) = f(1)$ but $-1 \neq 1$

$\therefore f$ is not one-one.

Consider $-1 \in R$

$f(x) = |x|$ is non-negative. There exist any element x in domain R such that $f(x) = |x| = -1$

$\therefore f$ is not onto.

The modulus function is neither one-one nor onto.

Question 5:

Show that the signum function $f: R \to R$ given by $f(x)=\begin{cases} 1, \text{ if } x>0 \\ 0, \text{ if } x=0 \\ -1, \text{ if } x<0 \end{cases}$ is neither one-one nor onto.

Solution:

$f: R \to R$ is $f(x)=\begin{cases} 1, \text{if } x>0 \\ 0, \text{ if } x=0 \\ -1, \text{ if } x<0 \end{cases}$

$f(1)=f(2)=1$, but $1 \neq 2$

$\therefore f$ is not one-one.

$f(x)$ takes only 3 values $(1,0,-1)$ for the element -2 in co-domain R, there does not exist any x in domain R such that $f(x)=-2$.

$\therefore f$ is not onto.

The signum function is neither one-one nor onto.

Question 6:

Let $A=\{1,2,3\}$, $B=\{4,5,6,7\}$ and let $f=\{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that f is one-one.

Solution:

$A=\{1,2,3\}$, $B=\{4,5,6,7\}$

$f: A \to B$ is defined as $f=\{(1,4),(2,5),(3,6)\}$

$\therefore f(1)=4, f(2)=5, f(3)=6$

It is seen that the images of distinct elements of A under f are distinct.

$\therefore f$ is one-one.

Question 7:

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

i. $f: R \to R$ defined by $f(x)=3-4x$

ii. $f: R \to R$ defined by $f(x)=1+x^2$

Solution:

i. $f: R \to R$ defined by $f(x) = 3 - 4x$

$x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow -4x = -4x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one.

For any real number (y) in R, there exists $\frac{3-y}{4}$ in R such that $f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y$

$\therefore f$ is onto.

Hence, f is bijective.

ii. $f: R \to R$ defined by $f(x) = 1 + x^2$

$x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

Consider $f(1) = f(-1) = 2$

$\therefore f$ is not one-one.

Consider an element -2 in co domain R.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in R$.

$\therefore f$ is not onto.

Hence, f is neither one-one nor onto.

Question 8:

Let A and B be sets. Show that $f: A \times B \to B \times A$ such that $(a, b) = (b, a)$ is a bijective function.

Solution:

$f: A \times B \to B \times A$ is defined as $(a, b) = (b, a)$.

$(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\therefore f$ is one-one.

$(b, a) \in B \times A$ there exist $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$

$\therefore f$ is onto.

f is bijective.

Question 9:

Let $f : N \to N$ be defined as $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{if } n \text{ is even} \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

Solution:

$f : N \to N$ be defined as $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{if } n \text{ is even} \end{cases}$ for all $n \in N$.

$f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$

$f(1) = f(2)$, where $1 \neq 2$

$\therefore f$ is not one-one.

Consider a natural number n in co domain N.

Case I: n is odd

$\therefore n = 2r + 1$ for some $r \in N$ there exists $4r + 1 \in N$ such that

$f(4r+1) = \frac{4r+1+1}{2} = 2r + 1$

Case II: n is even

$\therefore n = 2r$ for some $r \in N$ there exists $4r \in N$ such that

$f(4r) = \frac{4r}{2} = 2r$

$\therefore f$ is onto.

f is not a bijective function.

Question 10:

Let $A = R - \{3\}, B = R - \{1\}$ and $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Solution:

$A = R - \{3\}, B = R - \{1\}$ and $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$

$x, y \in A$ such that $f(x) = f(y)$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$
$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$
$$\Rightarrow -3x - 2y = -3y - 2x$$
$$\Rightarrow 3x - 2x = 3y - 2y$$
$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Let $y \in B = R - \{1\}$, then $y \neq 1$

The function f is onto if there exists $x \in A$ such that $f(x) = y$.
Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1--y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

$\therefore f$ is onto.
Hence, the function is one-one and onto.

Question 11:

Let $f : R \to R$ defined as $f(x) = x^4$.Choose the correct answer.

A. f is one-one onto
B. f is many-one onto
C. f is one-one but not onto
D. f is neither one-one nor onto

Solution:

$f : R \to R$ defined as $f(x) = x^4$

$x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow x^4 = y^4$

$\Rightarrow x = \pm y$

$\therefore f(x) = f(y)$ does not imply that $x = y$.

For example $f(1) = f(-1) = 1$

$\therefore f$ is not one-one.

Consider an element 2 in co domain R there does not exist any x in domain R such that $f(x) = 2$.

$\therefore f$ is not onto.

Function f is neither one-one nor onto.
The correct answer is D.

Question 12:

Let $f : R \to R$ defined as $f(x) = 3x$.Choose the correct answer.

A. f is one-one onto
B. f is many-one onto
C. f is one-one but not onto
D. f is neither one-one nor onto

Solution:

$f : R \to R$ defined as $f(x) = 3x$

$x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

$\therefore f$ is one-one.

For any real number y in co domain R, there exist $\frac{y}{3}$ in R such that $f\left(\frac{y}{3}\right)=3\left(\frac{y}{3}\right)=y$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

The correct answer is A.