NCERT Solutions Class 12 Maths Chapter 1 Exercise 1.3

Question 1:

Let $f:\{1,3,4\}\rightarrow\{1,2,5\}$ and $g:\{1,2,5\}\rightarrow\{1,3\}$ be given by $f=\{(1,2),(3,5),(4,1)\}$ and $g=\{(1,3),(2,3),(5,1)\}$. Write down gof.

Solution:

The functions $f:\{1,3,4\}\rightarrow\{1,2,5\}$ and $g:\{1,2,5\}\rightarrow\{1,3\}$ are $f=\{(1,2),(3,5),(4,1)\}$ and $g=\{(1,3),(2,3),(5,1)\}$

$gof(1)=g[f(1)]=g(2)=3 \qquad [as\ f(1)=2\ and\ g(2)=3]$

$gof(3)=g[f(3)]=g(5)=1 \qquad [as\ f(3)=5\ and\ g(5)=1]$

$gof(4)=g[f(4)]=g(1)=3 \qquad [as\ f(4)=1\ and\ g(1)=3]$

$\therefore gof=\{(1,3),(3,1),(4,3)\}$

Question 2:

Let f,g,h be functions from R to R. Show that

$(f+g)oh=foh+goh$

$(f.g)oh=(foh).(goh)$

Solution:

$(f+g)oh=foh+goh$

$LHS=[(f+g)oh](x)$

$=(f+g)[h(x)]=f[h(x)]+g[h(x)]$

$=(foh)(x)+goh(x)$

$=\{(foh)+(goh)\}(x)=RHS$

$\therefore\{(f+g)oh\}(x)=\{(foh)+(goh)\}(x)\quad for\ all\ x\in R$

Hence, $(f+g)oh=foh+goh$

$(f.g)oh=(foh).(goh)$

$LHS=[(f.g)oh](x)$

$=(f.g)[h(x)]=f[h(x)].g[h(x)]$

$=(foh)(x).(goh)(x)$

$=\{(foh).(goh)\}(x)=RHS$

$\therefore[(f.g)oh](x)=\{(foh).(goh)\}(x)\quad for\ all\ x\in R$

Hence, $(f.g)oh=(foh).(goh)$

Question 3:

Find gof and fog, if

i. $f(x)=|x|$ and $g(x)=|5x-2|$

ii. $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

Solution:

i. $f(x)=|x|$ and $g(x)=|5x-2|$

$\therefore gof(x)=g(f(x))=g(|x|)=|5|x|-2|$

$fog(x)=f(g(x))=f(|5x-2|)=||5x-2||=|5x-2|$

ii. $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

$\therefore gof(x)=g(f(x))=g(8x^3)=(8x^3)^{\frac{1}{3}}=2x$

$fog(x)=f(g(x))=f\left(x^{\frac{1}{3}}\right)^3=8\left(x^{\frac{1}{3}}\right)^3=8x$

Question 4:

If $f(x)=\dfrac{(4x+3)}{(6x-4)}, x\neq\dfrac{2}{3}$, show that $fof(x)=x,\ \ for\ all\ x\neq\dfrac{2}{3}$. What is the reverse of f ?

Solution:

$$(fof)(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)$$

$$=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$$

$\therefore fof(x)=x \quad for\ all\ x\neq\dfrac{2}{3}$

$\Rightarrow fof=1$

Hence, the given function f is invertible and the inverse of f is f itself.

Question 5:

State with reason whether the following functions have inverse.

i. $f:\{1,2,3,4\}\to\{10\}$ with $f=\{(1,10),(2,10),(3,10),(4,10)\}$

ii. $g:\{5,6,7,8\}\to\{1,2,3,4\}$ with $g=\{(5,4),(6,3),(7,4),(8,2)\}$

iii. $h:\{2,3,4,5\}\to\{7,9,11,13\}$ with $h=\{(2,7),(3,9),(4,11),(5,13)\}$

Solution:

i. $f:\{1,2,3,4\}\to\{10\}$ with $f=\{(1,10),(2,10),(3,10),(4,10)\}$

f is a many one function as $f(1)=f(2)=f(3)=f(4)=10$

$\therefore f$ is not one-one.

Function f does not have an inverse.

ii. $g:\{5,6,7,8\}\to\{1,2,3,4\}$ with $g=\{(5,4),(6,3),(7,4),(8,2)\}$

g is a many one function as $g(5)=g(7)=4$

$\therefore g$ is not one-one.

Function g does not have an inverse.

iii. $h:\{2,3,4,5\}\to\{7,9,11,13\}$ with $h=\{(2,7),(3,9),(4,11),(5,13)\}$

All distinct elements of the set $\{2,3,4,5\}$ have distinct images under h.

$\therefore h$ is one-one.

h is onto since for every element y of the set $\{7,9,11,13\}$, there exists an element x in the set $\{2,3,4,5\}$, such that $h(x)=y$.

h is a one-one and onto function.

Function h has an inverse.

Question 6:

Show that $f:[-1,1]\to R$, given by $f(x)=\dfrac{x}{(x+2)}$ is one-one. Find the inverse of the function $f:[-1,1]\to Range\, f$.

(Hint: For $y\in Range\, f, y=f(x)=\dfrac{x}{x+2}$, for some x in $[-1,1]$, i,e., $x=\dfrac{2y}{(1-y)}$

Solution:

$f:[-1,1]\to R$, given by $f(x)=\frac{x}{(x+2)}$

For one-one

$f(x)=f(y)$

$\Rightarrow \frac{x}{x+2}=\frac{y}{y+2}$

$\Rightarrow xy+2x=xy+2y$

$\Rightarrow 2x=2y$

$\Rightarrow x=y$

$\therefore f$ is a one-one function.

It is clear that $f:[-1,1]\to R$ is onto.

$\therefore f:[-1,1]\to R$ is one-one and onto and therefore, the inverse of the function $f:[-1,1]\to R$ exists.

Let $g: Range\, f\to[-1,1]$ be the inverse of f.

Let y be an arbitrary element of range f.

Since $f:[-1,1]\to Range\, f$ is onto, we have:

$y=f(x)\ \ for\ same\ x\in[-1,1]$

$\Rightarrow y=\frac{x}{x+2}$

$\Rightarrow xy+2y=x$

$\Rightarrow x(1-y)=2y$

$\Rightarrow x=\frac{2y}{1-y}, y\neq 1$

Now, let us define $g: Range\, f\to[-1,1]$ as

$g(y)=\frac{2y}{1-y}, y\neq 1$

Now,

$$(gof)(x)=g(f(x))=g\left(\frac{x}{x+2}\right)=\frac{2\left(\frac{x}{x+2}\right)}{1-\frac{x}{x+2}}=\frac{2x}{x+2-x}=\frac{2x}{2}=x$$

$$(fog)(x)=f(g(y))=f\left(\frac{2y}{1-y}\right)=\frac{\frac{2y}{1-y}}{\frac{2y}{1-y}+2}=\frac{2y}{2y+2-2y}=\frac{2y}{2}=y$$

$$\therefore gof=I_{[-1,1]} \quad and \quad fog=I_{Range\ f}$$

$$\therefore f^{-1}=g$$

$$\Rightarrow f^{-1}(y)=\frac{2y}{1-y}, y\neq 1$$

Question 7:

Consider $f:R\rightarrow R$ given by $f(x)=4x+3$. Show that f is invertible. Find the inverse of f.

Solution:

$f:R\rightarrow R$ given by $f(x)=4x+3$

For one-one

$f(x)=f(y)$

$\Rightarrow 4x+3=4y+3$

$\Rightarrow 4x=4y$

$\Rightarrow x=y$

$\therefore f$ is a one-one function.

For onto

$y\in R,\ let\ y=4x+3$

$\Rightarrow x=\frac{y-3}{4}\in R$

Therefore, for any $y\in R$, there exists $x=\frac{y-3}{4}\in R$ such that

$f(x)=f\left(\frac{y-3}{4}\right)=4\left(\frac{y-3}{4}\right)+3=y$

$\therefore f$ is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g:R\rightarrow R$ by $g(x)=\frac{y-3}{4}$

Now,

$$(gof)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = x$$

$$(fog)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

$$\therefore gof = fog = I_R$$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{y-3}{4}.$$

Question 8:

Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Solution:

$f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$

For one-one:

Let $f(x) = f(y)$

$\Rightarrow x^2 + 4 = y^2 + 4$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y \qquad [as\ x \in R]$

$\therefore f$ is a one -one function.

For onto:

For $y \in [4, \infty),\ let\ y = x^2 + 4$

$\Rightarrow x^2 = y - 4 \geq 0 \qquad [as\ y \geq 4]$

$\Rightarrow x = \sqrt{y-4} \geq 0$

Therefore, for any $y \in R$, there exists $x = \sqrt{y-4} \in R$ such that

$$f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y$$

$\therefore f$ is an onto function.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g : [4, \infty) \to R_+$ by

$g(y)=\sqrt{y-4}$

Now, $gof(x)=g(f(x))=g(x^2+4)=\sqrt{(x^2+4)-4}=\sqrt{x^2}=x$

And $fog(y)=f(g(y))=f(\sqrt{y-4})=(\sqrt{y-4})^2+4=(y-4)+4=y$

$\therefore gof=fog=I_R$

Hence, f is invertible and the inverse of f is given by

$f^{-1}(y)=g(y)=\sqrt{y-4}$.

Question 9:

Consider $f:R_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left(\dfrac{(\sqrt{y+6})-1}{3}\right)$.

Solution:

$f:R_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$

Let y be an arbitrary element of $[-5,\infty)$.

Let $y=9x^2+6x-5$

$\Rightarrow y=(3x+1)^2-1-5$

$\Rightarrow y=(3x+1)^2-6$

$\Rightarrow (3x+1)^2=y+6$

$\Rightarrow 3x+1=\sqrt{y+6}$ $\quad [as\ y\geq -5\Rightarrow y+6>0]$

$\Rightarrow x=\dfrac{\sqrt{y+6}-1}{3}$

$\therefore f$ is onto, thereby range $f=[-5,\infty)$.

Let us define $g:[-5,\infty)\to R_+\ as\ g(y)=\dfrac{\sqrt{y+6}-1}{3}$

We have,

$$(gof)(x) = g(f(x)) = g(9x^2 + 6x - 5)$$
$$= g\left((3x+1)^2 - 6\right)$$
$$= \frac{\sqrt{(3x+1)^2 - 6 + 6} - 1}{3}$$
$$= \frac{3x+1-1}{3} = x$$

And,

$$(fog)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$
$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$$
$$= \left(\sqrt{y+6}\right)^2 - 6 = y + 6 - 6 = y$$
$$\therefore gof = I_R \quad and \quad fog = I_{[-5,\infty)}$$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}.$$

Question 10:

Let $f: X \to Y$ be an invertible function. Show that f has unique inverse.

(Hint: suppose g_1 and g_2 are two inverses of f. Then for all $y \in Y$, $fog_1(y) = I_Y(y) = fog_2(y)$. Use one-one ness of f.

Solution:

Let $f: X \to Y$ be an invertible function.

Also suppose f has two inverses (g_1 and g_2)

Then, for all $y \in Y$,

$$fog_1(y) = I_Y(y) = fog_2(y)$$
$$\Rightarrow f(g_1(y)) = f(g_2(y))$$
$$\Rightarrow g_1(y) = g_2(y) \qquad [f \text{ is invertible} \Rightarrow f \text{ is one-one}]$$
$$\Rightarrow g_1 = g_2 \qquad [g \text{ is one-one}]$$

Hence, f has unique inverse.

Question 11:

Consider $f:\{1,2,3\}\to\{a,b,c\}$ given by $f(1)=a, f(2)=b, f(3)=c$. Find $\left(f^{-1}\right)^{-1}=f$.

Solution:

Function $f:\{1,2,3\}\to\{a,b,c\}$ given by $f(1)=a, f(2)=b, f(3)=c$

If we define $g:\{a,b,c\}\to\{1,2,3\}$ as $g(a)=1, g(b)=2, g(c)=3$

$(fog)(a)=f(g(a))=f(1)=a$

$(fog)(b)=f(g(b))=f(2)=b$

$(fog)(c)=f(g(c))=f(3)=c$

And,

$(gof)(1)=g(f(1))=g(a)=1$

$(gof)(2)=g(f(2))=g(b)=2$

$(gof)(3)=g(f(3))=g(c)=3$

$\therefore gof=I_X$ and $fog=I_Y$ where $X=\{(1,2,3)\}$ and $Y=\{a,b,c\}$

Thus, the inverse of f exists and $f^{-1}=g$.

$\therefore f^{-1}:\{a,b,c\}\to\{1,2,3\}$ is given by, $f^{-1}(a)=1, f^{-1}(b)=2, f^{-1}(c)=3$

We need to find the inverse of f^{-1} i.e., inverse of g.

If we define $h:\{1,2,3\}\to\{a,b,c\}$ as $h(1)=a, h(2)=b, h(3)=c$

$(goh)(1)=g(h(1))=g(a)=1$

$(goh)(2)=g(h(2))=g(b)=2$

$(goh)(3)=g(h(3))=g(c)=3$

And,

$(hog)(a)=h(g(a))=h(1)=a$

$(hog)(b)=h(g(b))=h(2)=b$

$(hog)(c)=h(g(c))=h(3)=c$

$\therefore goh=I_X$ and $hog=I_Y$ where $X=\{(1,2,3)\}$ and $Y=\{a,b,c\}$

Thus, the inverse of g exists and $g^{-1} = h \Rightarrow \left(f^{-1}\right)^{-1} = h$.

It can be noted that $h = f$.

Hence, $\left(f^{-1}\right)^{-1} = f$

Question 12:

Let $f : X \rightarrow Y$ be an invertible function. Show that the inverse of f^{-1} is f i.e., $\left(f^{-1}\right)^{-1} = f$.

Solution:

Let $f : X \rightarrow Y$ be an invertible function.

Then there exists a function $g : Y \rightarrow X$ such that $gof = \mathrm{I}_X$ and $fog = \mathrm{I}_Y$

Here, $f^{-1} = g$

Now, $gof = \mathrm{I}_X$ and $fog = \mathrm{I}_Y$

$\Rightarrow f^{-1}of = \mathrm{I}_X$ and $fof^{-1} = \mathrm{I}_Y$

Hence, $f^{-1} : Y \rightarrow X$ is invertible and f^{-1} is f i.e., $\left(f^{-1}\right)^{-1} = f$.

Question 13:

If $f : R \rightarrow R$ is given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then $fof(x)$ is:

A. $\frac{1}{x^3}$
B. x^3
C. x
D. $\left(3 - x^3\right)$

Solution:

$f : R \rightarrow R$ is given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$

$f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$

$$\therefore fof(x) = f\left(f(x)\right) = f\left(\left(3 - x^3\right)^{\frac{1}{3}}\right) = \left[3 - \left(\left(3 - x^3\right)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= \left[3 - \left(3 - x^3\right)\right]^{\frac{1}{3}} = \left(x^3\right)^{\frac{1}{3}} = x$$

$\therefore fof(x) = x$

The correct answer is C.

Question 14:

If $f: R-\left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x)=\frac{4x}{3x+4}$. The inverse of f is the map $g: Range\, f \to R-\left\{-\frac{4}{3}\right\}$ given by :

A. $g(y)=\frac{3y}{3-4y}$

B. $g(y)=\frac{4y}{4-3y}$

C. $g(y)=\frac{4y}{3-4y}$

D. $g(y)=\frac{3y}{4-3y}$

Solution:

It is given that $f: R-\left\{-\frac{4}{3}\right\} \to R$ is defined as $f(x)=\frac{4x}{3x+4}$

Let y be an arbitrary element of Range f.

Then, there exists $x \in R-\left\{-\frac{4}{3}\right\}$ such that $y=f(x)$.

$\Rightarrow y=\frac{4x}{3x+4}$

$\Rightarrow 3xy+4y=4x$

$\Rightarrow x(4-3y)=4y$

$\Rightarrow x=\frac{4y}{4-3y}$

Define $f: R-\left\{-\frac{4}{3}\right\} \to R$ as $g(y)=\frac{4y}{4-3y}$

Now,

$$(gof)(x) = g(f(x)) = g\left(\frac{4x}{3x+4}\right)$$

$$= \frac{4\left(\frac{4x}{3x+4}\right)}{4-3\left(\frac{4x}{3x+4}\right)} = \frac{16x}{12x+16-12x}$$

$$= \frac{16x}{16} = x$$

And

$$(fog)(x) = (g(x)) = f\left(\frac{4y}{4-3y}\right)$$

$$= \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4} = \frac{16y}{12y+16-12y}$$

$$= \frac{16y}{16} = y$$

$\therefore gof = I_{R-\left\{-\frac{4}{3}\right\}}$ and $fog = I_{Range\ f}$

Thus, g is the inverse of f i.e., $f^{-1} = g$

Hence, the inverse of f is the map $g : Range\ f \to R - \left\{-\frac{4}{3}\right\}$, which is given by $g(y) = \frac{4y}{4-3y}$.

The correct answer is B.