Chapter 11: Constructions

Exercise 11.1 (Page 219 of Grade 10 NCERT Textbook)

In each of the following, give the justification of the construction also:

Q1. Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts.

Difficulty level: Easy

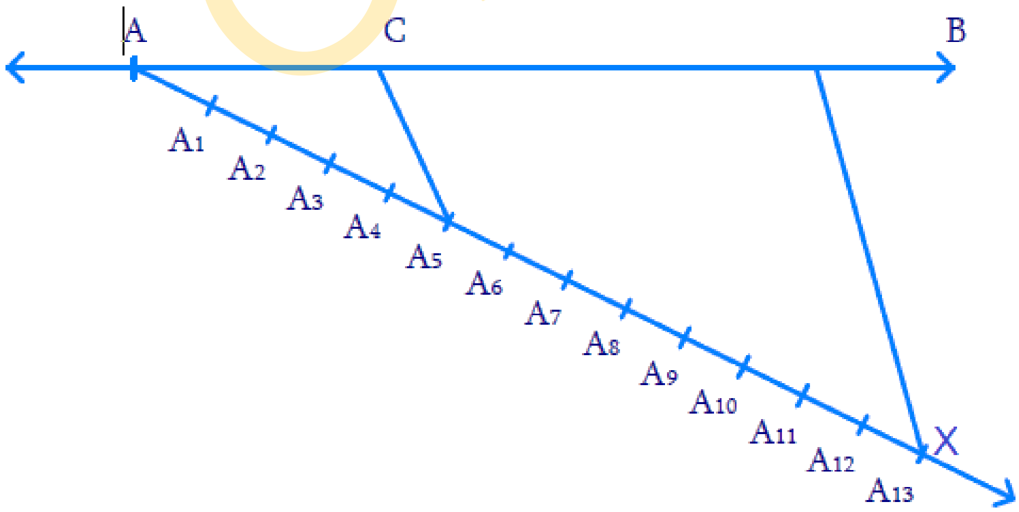
What is known/given?
Length of line segment and the ratio to be divided.

What is unknown?
Construction.

Reasoning:

- Draw the line segment of given length.
- Then draw another line which makes an acute angle with the given line.
- Divide the line into $m + n$ parts where m and n are the ratio given.
- Basic proportionality theorem states that, "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of construction:

(i) Draw $AB = 7.6$ cm
(ii) Draw ray AX, making an acute angle width AB.
(iii) Mark 13 (5 + 8) points $A_1, A_2, \ldots . A_{13}$ on AX such that
$AA_1 = A_1A_2 = A_2A_3 = \ldots\ldots A_{12}A_{13}$
(iv) Join BA_{13}
(v) Through A_5 (since we need 5 parts to 8 parts) draw CA_5 parallel to BA_{13} where C lies on AB.
Now $AC : CB = 5 : 8$
We find AC = 2.9 cm and CB = 4.7 cm

Proof:
CA_5 is parallel to BA_{13}
By Basic Proportionality theorem, in $\Delta AA_{13}B$

$$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}} = \frac{5}{8} \text{ (By Construction)}$$

Thus, C divides AB in the ratio 5:8.

Q2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Difficulty level: Medium

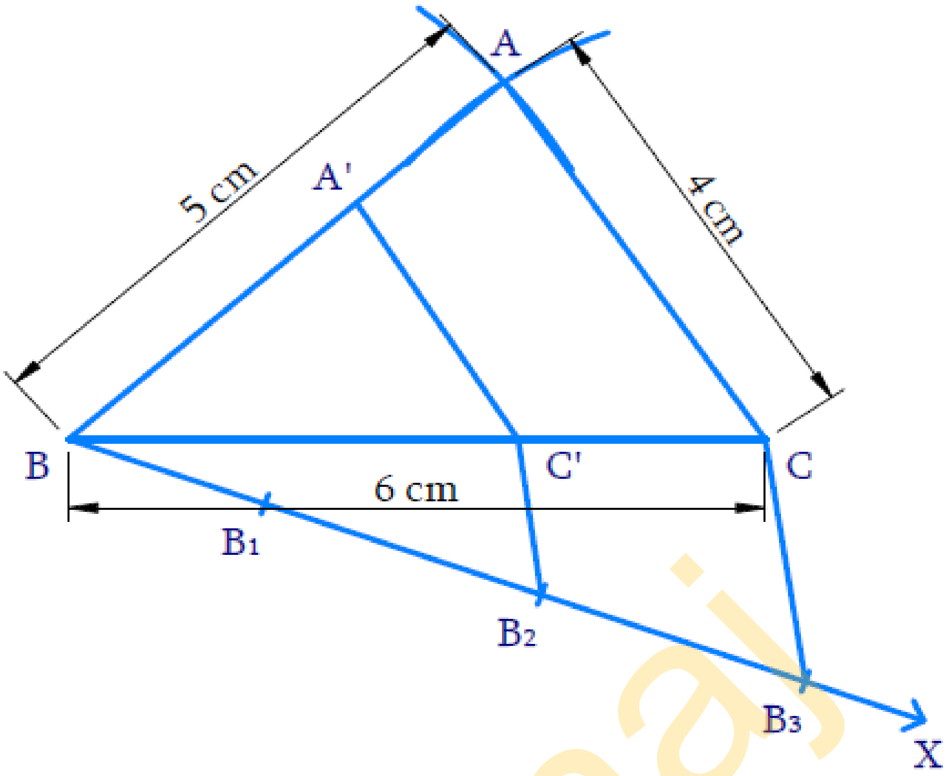
What is known/given?
Sides of the triangle and the ratio of corresponding sides of two triangles.

What is unknown?
Construction.

Reasoning:

- Draw the line segment of largest length 6 cm. Measure 5 cm and 4 cm separately and cut arcs from 2 ends of the line segment such that they cross each other at one point. Connect this point from both the ends.
- Then draw another line which makes an acute angle with the given line segment (6 cm).
- Divide the line into $(m + n)$ parts where m and n are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, “If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of constructions:

(i) Draw BC = 6 cm. With B and C as centres and radii 5 cm and 4 cm respectively draw arcs to intersect at A. ΔABC is obtained.

(ii) Draw ray BX making an acute angle with BC.

(iii) Mark 3 $\left(3 > 2 \text{ in the ratio } \frac{2}{3}\right)$ points B_1, B_2, B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$.

(iv) Join B_3C and draw the line through B_2 $\left(\text{2nd point where } 2 < 3 \text{ in the ratio } \frac{2}{3}\right)$ parallel to B_3C meeting BC at C'.

(v) Draw a line thorough C' parallel to CA to meet BA at A'. Now $\Delta A'BC'$ is the required triangle similar to ΔABC where

$$\frac{BC'}{BC} = \frac{BA'}{BA} = \frac{C'A'}{CA} = \frac{2}{3}$$

Proof:

In ΔBB_3C, B_2C' is parallel to B_3C.

Hence by Basic proportionality theorem,

$$\frac{B_2B_3}{BB_2} = \frac{C'C}{BC'} = \frac{1}{2}$$

Adding 1,

$$\frac{C'C}{BC'}+1=\frac{1}{2}+1$$

$$\frac{C'C+BC'}{BC'}=\frac{3}{2}$$

$$\frac{BC}{BC'}=\frac{3}{2}$$

$$\text{(or)}\quad \frac{BC'}{BC}=\frac{2}{3} \qquad \ldots(1)$$

Consider $\Delta BA'C'$ and ΔBAC

$$\angle A'BC'=\angle ABC \qquad (\text{Common})$$

$$\angle BA'C'=\angle BAC \qquad (\text{Corresponding angles } \because C'A' \parallel CA)$$

$$\angle BA'C'=\angle BCA \qquad (\text{Corresponding angles } \because C'A' \parallel CA)$$

Hence by AAA axiom, $\Delta BA'C' \sim \Delta BAC$

Corresponding sides are proportional

$$\frac{BA'}{BA}=\frac{C'A'}{CA}=\frac{BC'}{BC}=\frac{2}{3}\ (\text{from}\,(1))$$

Q3. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Difficulty level: Medium

What is known/given?

Sides of the triangle and the ratio of corresponding sides of 2 triangles.

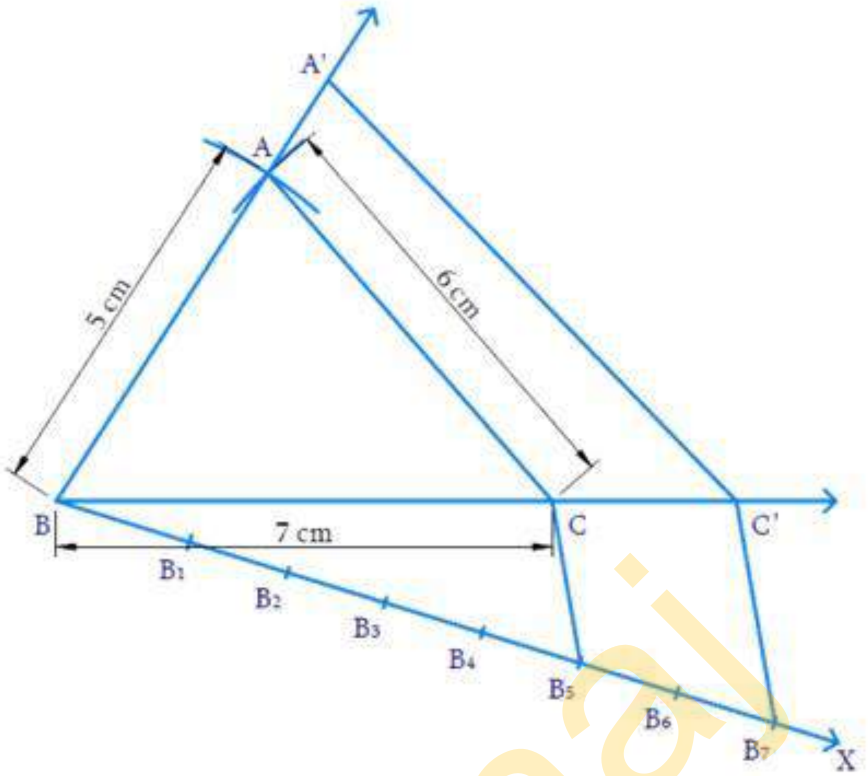
What is unknown?

Construction.

Reasoning:

- Draw the line segment of largest length 7 cm. Measure 5 cm and 6 cm separately and cut arcs from 2 ends of the line segment such that they cross each other at one point. Connect this point from both the ends.
- Then draw another line which makes an acute angle with the given line (7 cm). Divide the line into $m + n$ parts where m and n are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of construction:

(i) Draw $BC = 7\text{cm}$ with B and C as centres and radii 5 cm and 6 cm respectively. Draw arcs to intersect at A. ΔABC is obtained.

(ii) Draw ray BX making $\angle CBX$ acute.

(iii) Mark 7 points (greater of 7 and 5 in $\frac{7}{5}$) $B_I, B_2, \ldots\ldots\ldots B_7$ on BX such that

$BB_I = B_IB_2 = \ldots\ldots\ldots\ldots = B_6B_7$

(iv) Join B_5 $\left(\text{smaller of 7 and 5 in} \frac{7}{5} \text{ and so the } 5^{th} \text{point}\right)$ to C and draw B_7C' parallel to B_5C intersecting the extension of BC at C'.

(v) Through C' draw $C'A'$ parallel to CA to meet the extension of BA at A'. Now, $\Delta A'B'C'$ is the required triangle similar to ΔABC where

$$\frac{BA'}{BA} = \frac{C'A'}{CA} = \frac{BC'}{BC} = \frac{7}{5}$$

Proof:

In $\Delta BB_7C'$, B_3C is parallel to B_7C'

Hence by Basic proportionality theorem,

$$\frac{B_6B_7}{BB_5} = \frac{CC'}{BC} = \frac{2}{5}$$

Adding 1,

$$\frac{CC'}{BC}+1=\frac{2}{5}+1$$
$$\frac{BC+CC'}{BC}=\frac{7}{5}$$
$$\frac{BC'}{BC}=\frac{7}{5}$$

Consider ΔBAC and $\Delta BA'C'$

$$\angle ABC = \angle A'BC' \quad (\text{Common})$$
$$\angle BCA = \angle BC'A' \quad (\text{Corresponding angles} \quad \because CA \parallel C'A')$$
$$\angle BAC = \angle BA'C' \quad (\text{Corresponding angles})$$

By AAA axiom, $\Delta BAC \sim \Delta BA'C'$

$\therefore$ Corresponding sides are proportional

Hence,

$$\frac{BA'}{BA}=\frac{C'A'}{CA}=\frac{BC'}{BC}=\frac{7}{5}$$

Q4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Difficulty level: Medium

What is known/given?

Base and altitude of an isosceles triangle and the ratio of corresponding sides of 2 triangles.

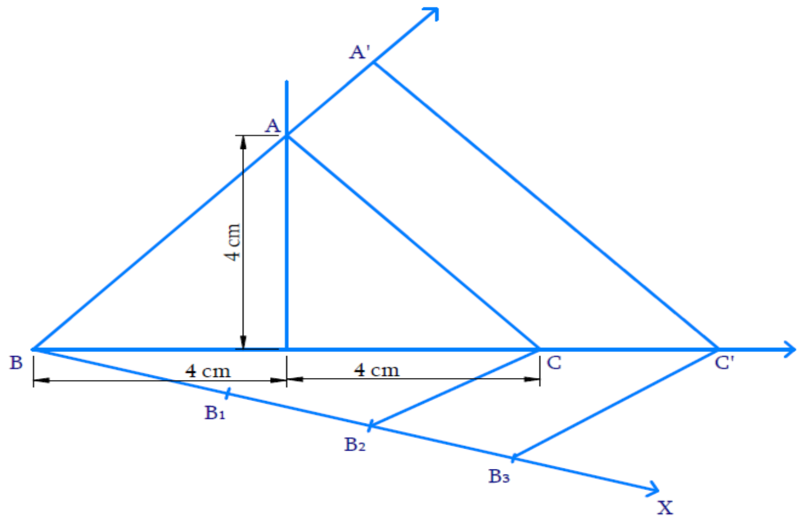
What is unknown?

Construction.

Reasoning:

- Draw the line segment of base 8 cm. Draw perpendicular bisector of the line. Mark a point on the bisector which measures 4 cm from the base. Connect this point from both the ends.
- Then draw another line which makes an acute angle with the given line. Divide the line into $m + n$ parts where m and n are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, “If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of construction:

(i) Draw $BC = 8cm$. Through D, the mid-point of BC, draw the perpendicular to BC and cut an arc from D on it such that $DA = 4cm$. Join BA and CA. ΔABC is obtained.

(ii) Draw the ray BX so that $\angle CBX$ is acute.

(iii) Mark 3 $\left(3 > 2 \text{ in } 1\frac{1}{2} = \frac{3}{2}\right)$ points B_1, B_2, B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$

(iv) Join B_2 $\left(2^{nd} \text{point } \because 2 < 3\right)$ to C and draw B_3C' parallel to B_2C, intersect BC extended at C'.

(v) Through C' draw $C'A'$ parallel to CA to intersect BA extended to A'.

Now, $\Delta A'BC'$ is the required triangle similar to ΔABC where

$$\frac{BA'}{BA} = \frac{C'A'}{CA} = \frac{BC'}{BC} = \frac{3}{2}$$

Proof:

In $\Delta BB_3C'$, $B_2C \parallel B_3C'$,

Hence by Basic proportionality theorem,

$$\frac{B_2B_3}{BB_2} = \frac{CC'}{BC} = \frac{1}{2}$$

Adding 1,

$$\frac{CC'}{BC} + 1 = \frac{1}{2} + 1$$

$$\frac{BC + CC'}{BC} = \frac{3}{2}$$

$$\frac{BC'}{BC} = \frac{3}{2}$$

Consider ΔBAC and $\Delta BA'C'$

$$\angle ABC = \angle A'BC' \qquad (\text{Common})$$

$$\angle BCA = \angle BC'A' \quad (\text{Corresponding angles } \because CA \parallel C'A')$$

$$\angle BAC = \angle BA'C' \quad (\text{Corresponding angles})$$

By AAA axiom, $\Delta BAC \sim \Delta BA'C'$

$\therefore$ Corresponding sides are proportional

Hence,

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{CA'}{CA} = \frac{3}{2}$$

Q5. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^0$. Then construct a triangle whose sides are $\frac{3}{2}$ of the corresponding sides of the triangle ABC.

Difficulty level: Medium

What is known/given?

2 sides and the angle between them and the ratio of corresponding sides of 2 triangles.

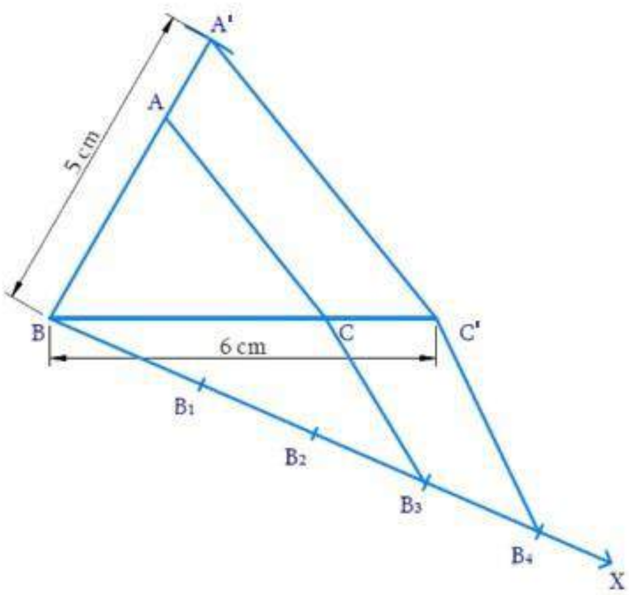
What is unknown?

Construction.

Reasoning:

- Draw the triangle with the given conditions.
- Then draw another line which makes an acute angle with the base line. Divide the line into $m + n$ parts where m and n are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of constructions:

(i) Draw $BC = 6$ cm. At B, make $\angle CBY = 60°$ and cut an arc at A so that BA = 5 cm. Join AC, ΔABC is obtained.

(ii) Draw the ray BX such that $\angle CBX$ is acute.

(iii) Mark 4 $\left(4 > 3 \text{ in} \frac{3}{4}\right)$ points B_1, B_2, B_3, B_4 on BX such that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(iv) Join B_4 to C and draw B_3C' parallel to B_4C to intersect BC at C'.

(v) Draw $C'A'$ parallel to CA to intersect BA at A'.

Now, $\Delta A'BC'$ is the required triangle similar to ΔABC where

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{3}{4}$$

Proof:

In $\Delta BB_4C'$, $B_3C' \parallel B_4C$

Hence by Basic proportionality theorem,

$$\frac{B_3B_4}{BB_3} = \frac{C'C}{BC'} = \frac{1}{3}$$

$$\frac{C'C}{BC'} + 1 = \frac{1}{3} + 1 \quad \text{(Adding 1)}$$

$$\frac{C'C + BC'}{BC'} = \frac{4}{3}$$

$$\frac{BC}{BC'} = \frac{4}{3} \quad \text{(or)} \quad \frac{BC'}{BC} = \frac{3}{4}$$

Consider $\Delta BA'C'$ and ΔBAC

$$\angle A'BC' = \angle ABC = 60^\circ$$

$$\angle BCA' = \angle BCA \quad (\text{Corresponding angles} \quad \because C'A' \parallel CA)$$

$$\angle BA'C' = \angle BAC \quad (\text{Corresponding angles})$$

By AAA axiom, $\Delta BA'C' \sim \Delta BAC$

Therefore, corresponding sides are proportional,

$$\frac{BC'}{BC} = \frac{BA'}{BA} = \frac{C'A'}{CA} = \frac{3}{4}$$

Q6. Draw a triangle ABC with side $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Difficulty level: Medium

What is known/given?

One side and 2 angles of a triangle and the ratio of corresponding sides of 2 triangles.

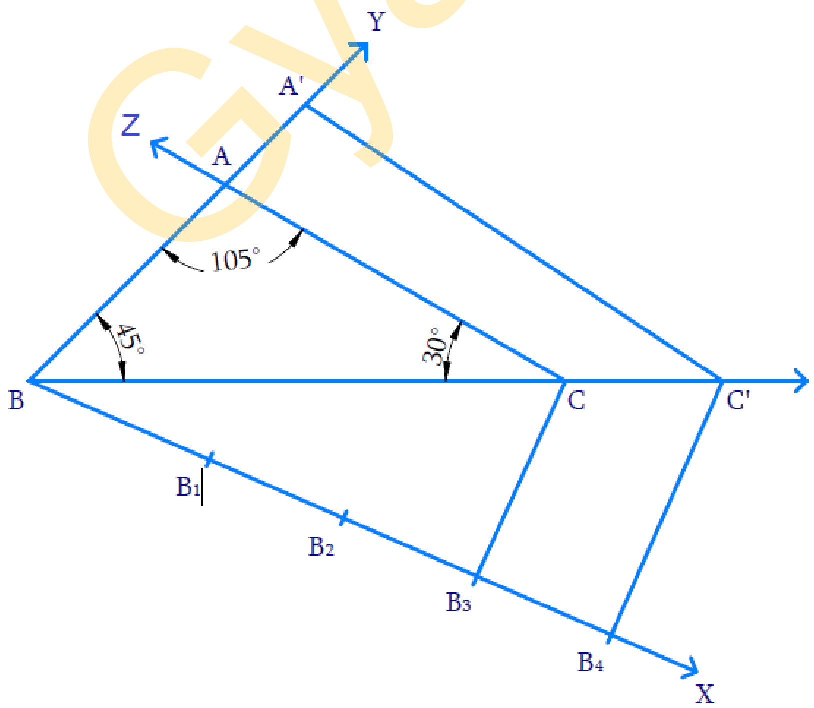
What is unknown?

Construction.

Reasoning:

- Draw the triangle with the given conditions.
- Then draw another line which makes an acute angle with the base line. Divide the line into *m* + *n* parts where *m* and *n* are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of construction:

(i) Draw $BC = 7\text{cm}.$ At B, make an angle $\angle CBY = 45^\circ$ and at C, make $\angle BCZ = 30^\circ\left[180^\circ - \left(45^\circ + 105^\circ\right)\right]$. Both BY and CZ intersect at A and thus ΔABC is constructed.

(ii) Draw the ray BX so that $\angle CBX$ is acute.

(iii) Mark 4 $\left(4 > 3 \text{ in } \frac{4}{3}\right)$ points B_1, B_2, B_3, B_4 on BX such that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(iv) Join B_3 $\left(\text{third point on BX, } 3 < 4 \text{ in } \frac{4}{3}\right)$ to C and draw B_4C' parallel to BC such that C' lies on the extension of BC.

(v) Draw C'A' parallel to CA to intersect the extension of BA at A'. Now, $\Delta A'BC'$ is the required triangle similar to ΔABC where,

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{4}{3}$$

Proof:

In $\Delta BB_4C'$, $B_3C \parallel B_4C'$

Hence by Basic proportionality theorem,

$$\frac{B_3B_4}{BB_3} = \frac{CC'}{BC} = \frac{1}{3}$$

$$\frac{CC'}{BC} + 1 = \frac{1}{3} + 1 \qquad \text{(Adding 1)}$$

$$\frac{BC + CC'}{BC} = \frac{4}{3}$$

$$\frac{BC'}{BC} = \frac{4}{3}$$

Consider $\Delta BA'C'$ and ΔBAC

$$\angle A'BC' = \angle ABC = 45^\circ$$

$$\angle BC'A' = \angle BCA = 30^\circ \qquad (\text{Corresponding angles as } CA \parallel C'A')$$

$$\angle BA'C' = \angle BAC = 105^\circ \qquad (\text{Corresponding angles})$$

By AAA axiom, $\Delta BA'C' \sim \Delta BAC$

Hence corresponding sides are proportional

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{4}{3}$$

Q7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

Difficulty level: Medium

What is known/given?

2 sides and the angle between them and the ratio of corresponding sides of 2 triangles.

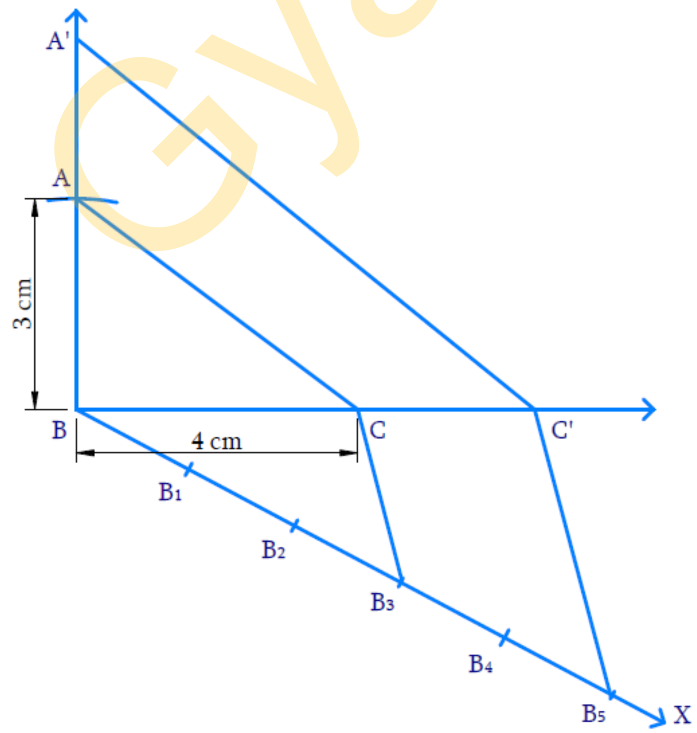
What is unknown?

Construction.

Reasoning:

- Draw the triangle with the given conditions.
- Then draw another line which makes an acute angle with the base line. Divide the line into $m + n$ parts where m and n are the ratio given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- Basic proportionality theorem states that, "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".

Solution:



Steps of constructions:

(i) Draw $BC = 4$ cm. At B, make an angle $\angle CBY = 90^\circ$ and mark A on BY such that $BA = 3$ cm. Join A to C. Thus ΔABC is constructed.

(ii) Draw the ray BX so that $\angle CBX$ is acute.

(iii) Mark 5 $\left(5 > 3 \text{ in } \frac{5}{3}\right)$ points B_1, B_2, B_3, B_4, B_5 on BX so that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

(iv) Join B_3 $\left(3^{rd} \text{point on BX as} 3 < 5\right)$ to C and draw B_5C' parallel to B_3C so that C' lies on the extension of BC.

(v) Draw $C'A'$ parallel to CA to intersect of the extension of BA at A'. Now $\Delta BA'C'$ is the required triangle similar to ΔBAC where $\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{5}{3}$

Proof:

In $\Delta BB_3C', B_3C \parallel B_3C'$

Hence by Basic proportionality theorem,

$$\frac{B_3B_5}{BB_3} = \frac{CC'}{BC} = \frac{2}{3}$$

$$\frac{CC'}{BC} + 1 = \frac{2}{3} + 1 \qquad \text{(Adding 1)}$$

$$\frac{CC' + BC}{BC} = \frac{5}{3}$$

$$\frac{BC'}{BC} = \frac{5}{3}$$

Consider ΔBAC and $\Delta BA'C'$

$$\angle ABC = \angle A'BC' = 90^\circ$$

$$\angle BCA = \angle BC'A' \qquad \text{(Corresponding angles as } CA \parallel C'A'\text{)}$$

$$\angle BAC = \angle BA'C'$$

By AAA axiom, $\Delta BAC \sim \Delta BA'C'$

Therefore, corresponding sides are proportional,

Hence,

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{5}{3}$$

Chapter 11: Constructions

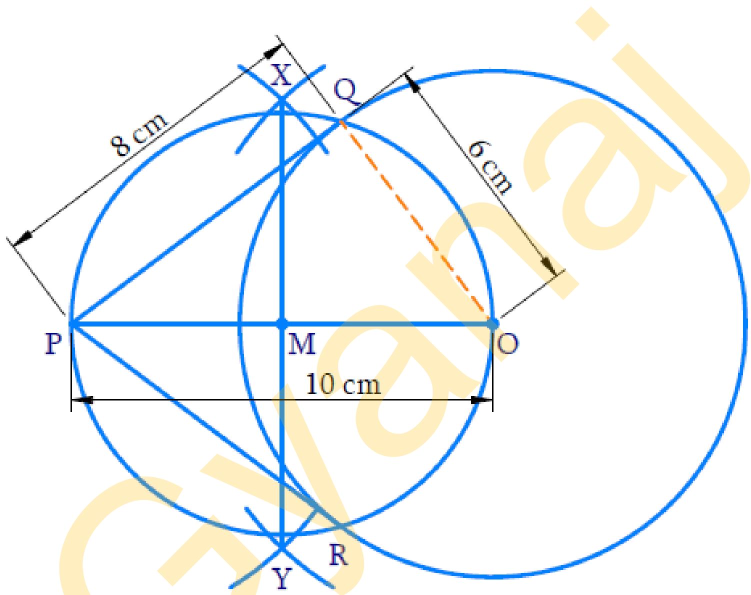
Exercise 11.2 (Page 221 of Grade 10 NCERT Textbook)

In each of the following, give also the justification of the construction:

Q1. Draw a circle of radius 6 cm. From a point 10 cm away from its center, construct the pair of tangents to the circle and measure their lengths.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Take a point O as centre and 6 cm radius. Draw a circle.
(ii) Take a point P such that OP = 10 cm.
(iii) With O and P as centres and radius more than half of OP draw arcs above and below OP to intersect at X and Y.
(iv) Join XY to intersect OP at M.
(v) With M as centre and OM as radius draw a circle to intersect the given circle at Q and R.
(vi) Join PQ and PR.

PQ and PR are the required tangents where $PQ = PR = 8$cm.

Proof:

$$\angle PQO = 90^\circ \Rightarrow PQ \perp OQ \quad (\text{Angle in a semicircle})$$

OQ being the radius of the given circle, PQ in the tangent at Q.

In right ΔPQO,

$$OP = 10 \text{ cm}, OQ = 6\text{cm} \ (\text{radius})$$

$$PQ^2 = OP^2 - OQ^2$$

$$= (10)^2 - (6)^2$$

$$= 100 - 36$$

$$= 64$$

$$PQ = \sqrt{64}$$

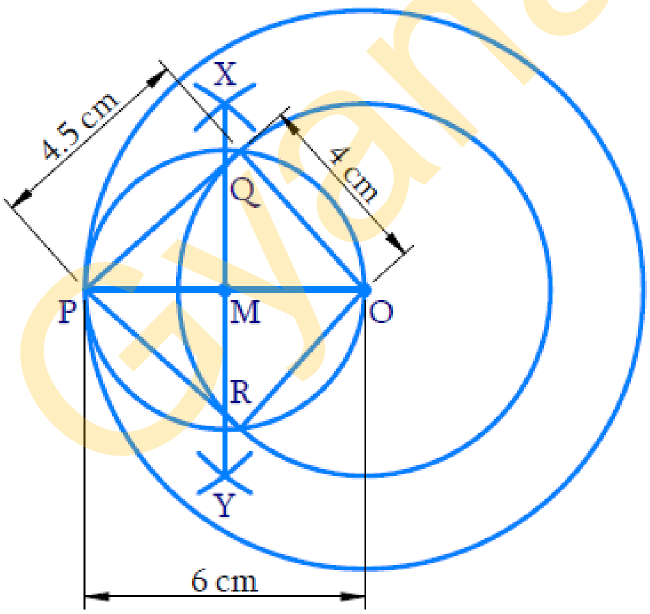
$$= 8\text{cm}$$

Similarly, PR = 8 cm.

Q2. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Take 'O' as centre and radius 4 cm and 6 cm draw two circles.
(ii) Take a point 'P' on the bigger circle and join OP.
(iii) With 'O' and 'P' as centre and radius more than half of OP draw arcs above and below OP to intersect at X and Y.
(iv) Join XY to intersect OP at M.
(v) With M as centre and *OM* as radius draw a circle to cut the smaller circle at Q and R.
(vi) Join PQ and PR.

PQ and PR are the required tangent where $PQ = 4.5\,(\text{aprox})$

Proof:

$$\angle PQO = 90^\circ \text{ (Angle in a semi-circle)}$$

$$\therefore PQ \perp OQ$$

OQ being the radius of the smaller circle, PQ is the tangent at Q.
In the right ΔPQO,

$$OP = 6 \text{ cm (radius of the bigger circle)}$$

$$OQ = 4 \text{ cm (radius of the smaller circle)}$$

$$PQ^2 = (OP)^2 - (OQ)^2$$
$$= (6)^2 - (4)^2$$
$$= 36 - 16$$
$$= 20$$

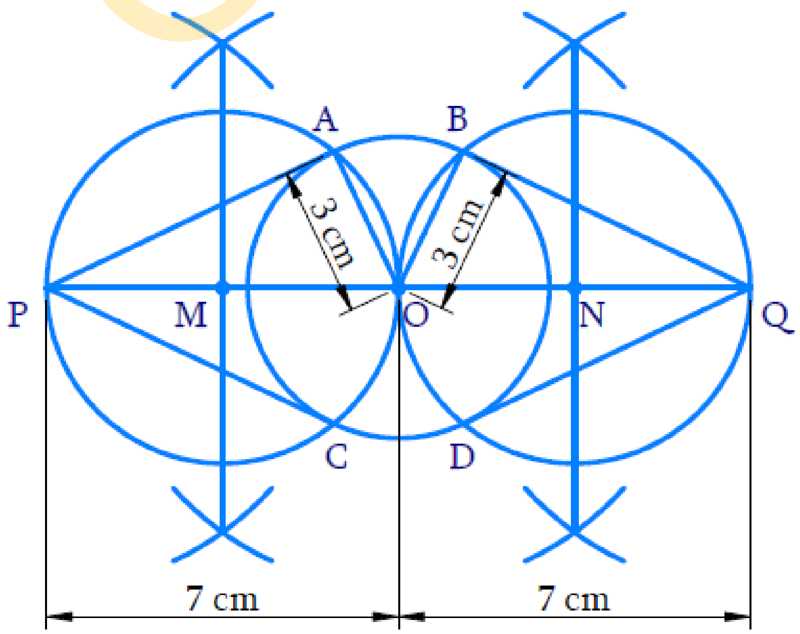
$$PQ = \sqrt{20}$$
$$= 4.5 \text{ (approx)}$$

Similarly, $PR = 4.5$ (approx.)

Q3. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameters each at a distance of 7 cm from its center. Draw tangents to the circle from these two points P and Q.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Draw a circle with O as centre and radius is 3 cm.

(ii) Draw a diameter of it extend both the sides and take points P, Q on the diameter such that $OP = OQ = 7$ cm.

(iii) Draw the perpendicular bisectors of OP and OQ to intersect OP and OQ at M and N respectively.

(iv) With M as centre and OM as radius draw a circle to cut the given circle at A and C. With N as centre and ON as radius draw a circle to cut the given circle at B and D.

(v) Join PA, PC, QB, QD

PA, PC and QB, QD are the required tangents from P and Q respectively.

Proof:

$$\angle PAO = \angle QBO = 90^\circ \left(\text{Angle in a semi-circle}\right)$$

$$\therefore PA \perp AO, \; QB \perp BO$$

Since OA and OB are the radii of the given circle, PA and QB are its tangents at A and B respectively.

In right angle triangle PAO and QBO

$$OP = OQ = 7\,\text{cm} \; \left(\text{By construction}\right)$$

$$OA = OB = 3\text{cm} \; \left(\text{radius of the given circle}\right)$$

$$PA^2 = (OP)^2 - (OA)^2$$
$$= (7)^2 - (3)^2$$
$$= 49 - 9$$
$$= 40$$
$$PA = \sqrt{40}$$
$$= 6.3 \text{ (approx)}$$

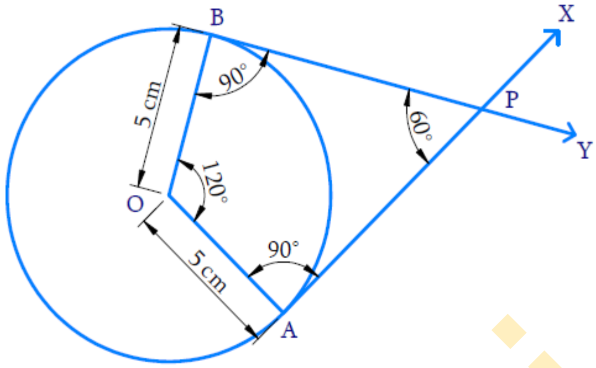
And

$$QB^2 = (OQ)^2 - (OB)^2$$
$$= (7)^2 - (3)^2$$
$$= 49 - 9$$
$$= 40$$
$$QB = \sqrt{40}$$
$$= 6.3 \text{ (approx)}$$

Q4. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Difficulty level: Medium

Solution:



Steps of construction:

(i) With O as centre and 5cm as radius draw a circle.
(ii) Take a point A on the circumference of the circle and join OA.
(iii) Draw AX perpendicular to OA.
(iv) Construct $\angle AOB = 120°$ where B lies on the circumference.
(v) Draw BY perpendicular to OB.
(vi) Both AX and BY intersect at P.
(vii) PA and PB are the required tangents inclined at $60°$.

Proof:

$$\angle OAP = \angle OBP = 90° \text{ (By construction)}$$
$$\angle AOB = 120° \text{ (By construction)}$$

In quadrilateral OAPB,

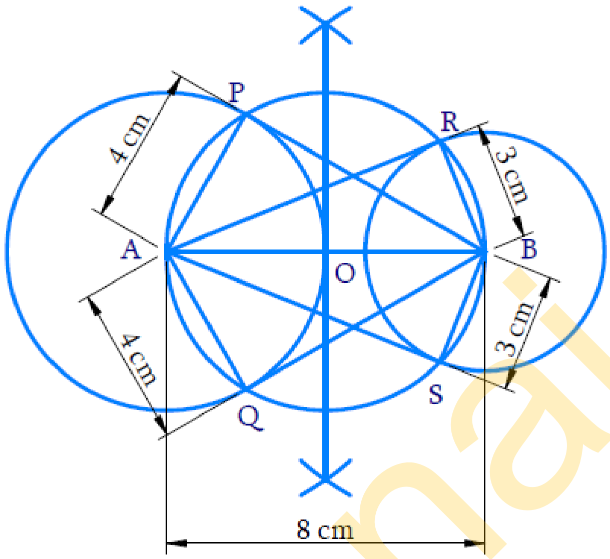
$$\begin{aligned}\angle APB &= 360° - [\angle OAP + \angle OBP + \angle AOB] \\ &= 360° - [90° + 90° + 120°] \\ &= 360° - 300° \\ &= 60°\end{aligned}$$

Hence PA and PB are the required tangents inclined at $60°$.

Q5. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as center, draw another circle of radius 3 cm. Construct tangent to each circle from the center of the other circle.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Draw $AB = 8\text{cm}$. With A and B as centers 4 cm and 3 cm as radius respectively draw two circles.

(ii) Draw the perpendicular bisector of AB, intersecting AB at O.

(iii) With *O* as center and OA as radius draw a circle which intersects the two circles at P, Q, R and S.

(iv) Join BP, BQ, AR and AS.

(v) BP and BQ are the tangents from B to the circle with center A. AR and AS are the tangents from A to the circle with center B.

Proof:

$$\angle APB = \angle AQB = 90^\circ \text{ (Angle in a semi-circle)}$$

$$\therefore AP \perp PB \text{ and } AQ \perp QB$$

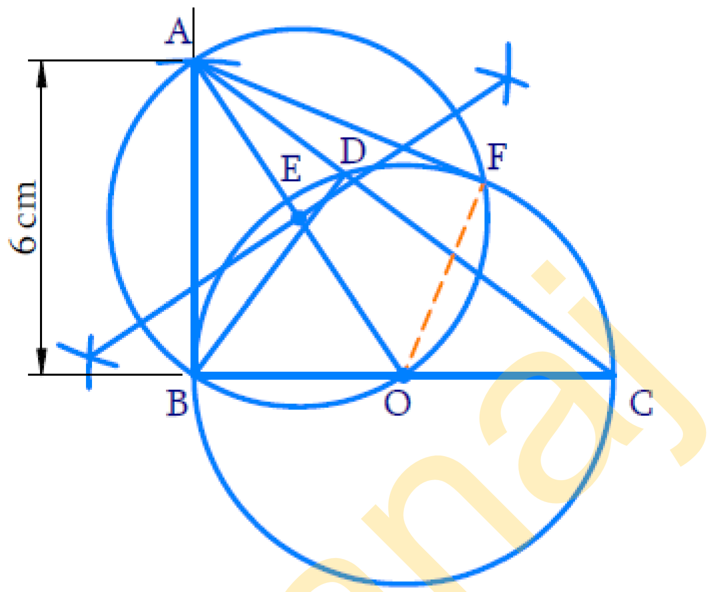
Therefore, BP and BQ are the tangents to the circle with center A.

Similarly, AR and AS are the tangents to the circle with center B.

Q6. Let *ABC* be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90°$. BD is the perpendicular to AC. The circle through B, C and D is drawn. Construct the tangents from A to this circle.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Draw BC = 8 cm. Draw the perpendicular at B and cut BA = 6 cm on it. Join AC right ΔABC is obtained.

(ii) Draw BD perpendicular to AC.

(iii) Since $\angle BDC = 90°$ and the circle has to pass through B, C and D. BC must be a diameter of this circle. So, take O as the midpoint of BC and with O as centre and OB as radius draw a circle which will pass through B, C and D.

(iv) To draw tangents from *A* to the circle with center O.

a) Join OA, and draw its perpendicular bisectors to intersect OA at E.

b) With E as center and EA as radius draw a circle which intersects the previous circle at B and F.

c) Join AF.

AB and AF are the required tangents.

Proof:

$$\angle ABO = \angle AFO = 90° \qquad (\text{Angle in a semi-circle})$$

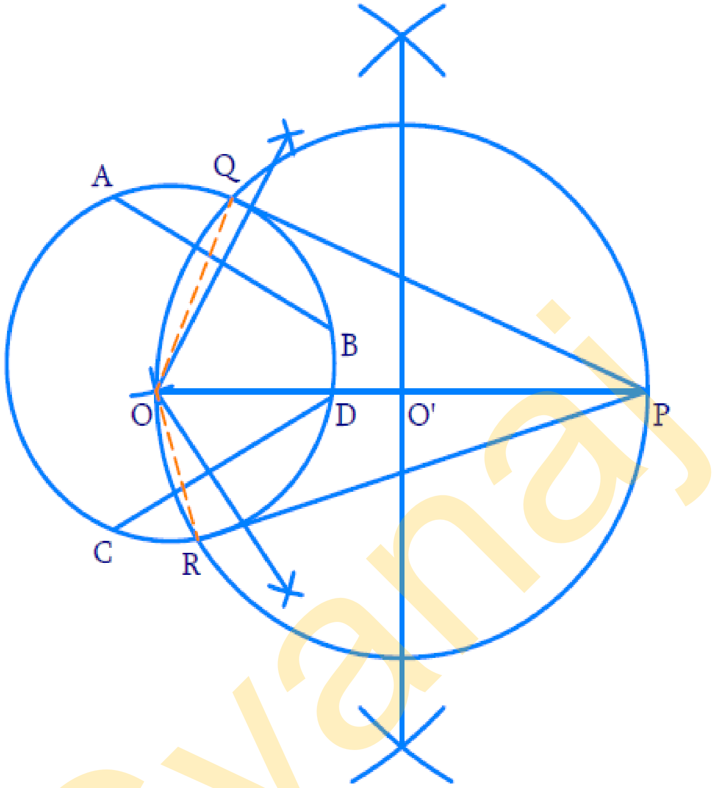
$$\therefore AB \perp OB \text{ and } AF \perp OF$$

Hence AB and AF are the tangents from A to the circle with centre O.

Q7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

Difficulty level: Medium

Solution:



Steps of construction:

(i) Draw any circle using a bangle.
To find its centre

(a) Draw any two chords of the circle say AB and CD.
(b) Draw the perpendicular bisectors of AB and CD to intersect at O.

Now, 'O' is the centre of this circle (since the perpendiculars drawn from the centre of a circle to any chord bisect the chord and vice versa).

To draw the tangents from a point 'P' outside the circle.

(ii) Take any point P outside the circle and draw the perpendicular bisector of OP which meets at OP at O'.

(iii) With O' as center and OO' as radius draw a circle which cuts the given circle at Q and R.

(iv) Join PQ and PR.

PQ and PR are the required tangents.

Proof:

$\angle OQP = \angle ORP = 90^\circ$ (Angle in a semi-circle)

$\therefore OQ \perp QP$ and $OR \perp RP$.

Hence, PQ and PR are the tangents to the given circle.