Chapter - 2: Polynomials

Exercise 2.1 (Page 28 of Grade 10 NCERT Textbook)

Q1. The graphs of $y = p(x)$ are given in the figure below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

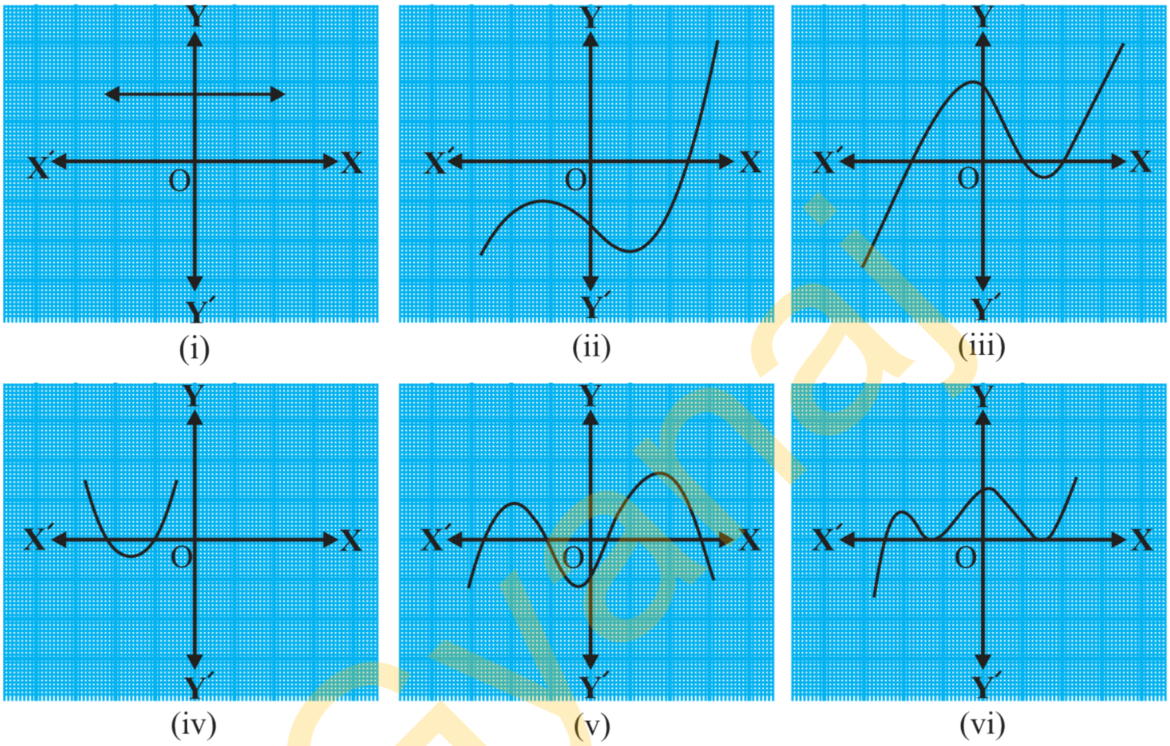

Fig. 2.10

Difficulty Level: Easy

What is given /known?

The graphs of $y = p(x)$ are given in the above figure for some polynomials $p(x)$.

What is the unknown?

The number of zeroes of $p(x)$, in each case.

Reasoning:

You can reach the solution easily by understanding the statement of the question. As the graphs of $y = p(x)$ are given, and you have to find the number of zeroes of $p(x)$, in each case.

To find this, look at the graphs and visually find how many points the graph cuts or touches the *x*–axis. The number of points it cuts or touches the *x*–axis are the zeroes of the polynomial $p(x)$.

Solution:

(i) The number of zeroes is 0 as the graph doesn't cuts the x–axis at any point.

(ii) The number of zeroes is 1 as the graph cuts the x–axis at only one point.

(iii) The number of zeroes is 3 as the graph cuts the x–axis at 3 points.

(iv) The number of zeroes is 2 as the graph cuts the x–axis at 2 points.

(v) The number of zeroes is 4 as the graph cuts the x–axis at 4 points.

(vi) The number of zeroes is 3 as the graph cuts the x–axis at 3 points.

Chapter - 2: Polynomials

Exercise 2.2 (Page 33 of Grade 10 NCERT Textbook)

Q1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Difficulty Level: Medium

What is given /known?
Quadratic polynomials.

What is the unknown?

- The zeroes of the given quadratic polynomials.
- Verification of the relationship between the zeroes and the coefficients.

Reasoning:
You can solve this question by following the steps given below:

We know that the standard form of the quadratic equation is:

$$ax^2 + bx + c = 0$$

Simplify the quadratic polynomial by factorisation and find the zeroes of the polynomial.

Now you have to find the relation between the zeroes and the coefficients.

For find out the sum of zeroes and product of zeroes.

We know that

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\text{Product of Zeroes} = \frac{constant\ term}{coefficient\ of\ x^2}$$

$$\alpha . \beta = \frac{c}{a}$$

Put the values in the above formula and find out the relation between the zeroes and the coefficients.

Solution:

i.

$$x^2 - 2x - 8$$
$$x^2 - 4x + 2x - 8 = 0$$
$$x(x - 4) + 2(x - 4) = 0$$
$$(x-4)(x+2) = 0$$
$$(x-4) = 0,$$
$$(x + 2) = 0$$

$x = 4$, $x = -2$ are the zeroes of the polynomial.

Relationship between the zeroes and the coefficients

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$
$$\alpha + \beta = \frac{-b}{a}$$
$$-2 + 4 = \frac{-(-2)}{1}$$
$$2 = 2$$

$$\text{Product of zeroes} = \frac{constant\ term}{coefficient\ of\ x^2}$$
$$\alpha . \beta = \frac{c}{a}$$
$$-2 \times 4 = \frac{-8}{1}$$
$$-8 = -8$$

ii.

$$4s^2 - 4s + 1$$
$$4s^2 - 2s - 2s + 1 = 0$$
$$2s(2s - 1) - (2s - 1) = 0$$
$$(2s - 1)(2s - 1) = 0$$
$$2s - 1 = 0, 2s - 1 = 0$$

$s = \frac{1}{2}, s = \frac{1}{2}$ are the zeroes of the polynomial.

Relationship between the zeroes and the coefficients

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$$

$$1 = 1$$

$$\text{Product of zeroes} = \frac{constant\ term}{coefficient\ of\ x^2}$$

$$\alpha . \beta = \frac{c}{a}$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

iii.

$$6x^2 - 3 - 7x$$

$$6x^2 - 7x - 3 = 0$$

$$6x^2 - 9x + 2x - 3 = 0$$

$$3x(2x - 3) + (2x - 3) = 0$$

$$(2x - 3) = 0,\ (3x + 1) = 0$$

$x = \frac{3}{2},\ x = \frac{-1}{3}$ are the zeroes of the polynomial

Relationship between the zeroes and the coefficients:

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

$$\alpha + \beta = \frac{-(-7)}{6}$$

$$\frac{3}{2} + \frac{-1}{3} = \frac{(7)}{6}$$

$$\frac{(7)}{6} = \frac{(7)}{6}$$

$$\alpha.\beta=\frac{c}{a}$$

$$\frac{3}{2}\times\frac{-1}{3}=\frac{(-3)}{6}$$

$$\frac{(-3)}{6}=\frac{(-3)}{6}$$

$$\frac{(-1)}{2}=\frac{(-1)}{2}$$

iv.

$$4u^2+8u$$

$$4u(u+2)=0$$

$$4u=0 \text{ or } u+2=0$$

$$u=0 \text{ or } u=-2$$

$u=0,\ u=-2$ are the zeroes of the polynomial

Relationship between the zeroes and the coefficients

$$\text{Sum of zeroes}=\frac{-coefficient\ of\ u}{coefficient\ of\ u^2}$$

$$\alpha+\beta=\frac{-(8)}{4}$$

$$0+(-2)=-2$$

$$-2=-2$$

$$\text{Product of zeroes}=\frac{constant\ term}{coefficient\ of\ u^2}$$

$$\alpha.\beta=\frac{c}{a}$$

$$0\times-2=\frac{0}{4}$$

$$0=0$$

v.

$$t^2 - 15$$
$$t^2 - 15 = 0$$
$$t^2 - 15 = 0$$
$$t = \sqrt{15}$$

$-\sqrt{15}, +\sqrt{15}$ are the zeroes of the polynomial

Relationship between the zeroes and the coefficients

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ t}{coefficient\ of\ t^2}$$
$$\alpha + \beta = \frac{0}{1}$$
$$= -\sqrt{15} + \sqrt{15} = 0$$
$$0 = 0$$

$$\text{Product of zeroes} = \frac{constant\ term}{coefficient\ of\ t^2}$$
$$\alpha . \beta = \frac{c}{a}$$
$$-\sqrt{15} \times \sqrt{15} = -\frac{15}{1}$$
$$-15 = -15$$

vi.

$$3x^2 - x - 4$$
$$3x^2 - x - 4 = 0$$
$$3x^2 - 4x + 3x - 4 = 0$$
$$x(3x-4) + (3x-4) = 0$$
$$(x+1)(3x-4) = 0$$
$$(x+1) = 0 \quad \text{or} \quad (3x-4) = 0$$

$x = -1$ or $x = \frac{4}{3}$ are the zeroes of the polynomial

Relationship between the zeroes and the coefficients

$$\text{Sum of zeroes} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

$$\alpha + \beta = \frac{-1}{3}$$

$$-1 + \frac{4}{3} = \frac{-1}{3}$$

$$\frac{1}{3} = \frac{1}{3}$$

$$\text{product of zeroes} = \frac{\textit{constant term}}{\textit{coefficient of } x^2}$$

$$\alpha . \beta = \frac{c}{a}$$

$$-1 \times \frac{4}{\frac{4}{3}} = -\frac{4}{3}$$

$$-\frac{4}{3} = -\frac{4}{3}$$

Q2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) 0, 5

(iv) 1, 1 (v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4,1

Difficulty Level: Medium

What is given /known?

The sum and product of zeroes of quadratic polynomials.

What is the unknown?

A quadratic polynomial with the given numbers as the sum and product of its zeroes respectively.

Reasoning:

This question is straight forward - the value of sum of roots and product of roots is given. You have to form a quadratic polynomial. Put the values in the general equation of the quadratic polynomial i.e.

$$K\left(x^2 - (\text{sum of roots})x + \text{product of roots}\right)$$

Solution:

(i) $\frac{1}{4},-1$

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2-(\text{sum of roots})\ x+\ \text{product of roots}\ \right)$$
$$k\left\{x^2-\frac{1}{4}x+\frac{1}{4}\times-1\right\}$$
$$k\left\{x^2-\frac{1}{4}x-\frac{1}{4}\right\}$$

(ii) $\sqrt{2},\frac{1}{3}$

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2-(\text{sum of roots})x+\text{product of roots}\right)$$
$$k\left\{x^2-\sqrt{2}x+\frac{1}{3}\right\}$$

(iii) $0,\sqrt{5}$

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2-\ (\text{sum of roots})x\ +\ \text{product of roots}\right)$$
$$k\left\{x^2-\ 0.x\ +\ \sqrt{5}\right\}$$
$$k\left\{x^2\ +\ \sqrt{5}\right\}$$

(iv) $1,1$

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2-\ (\text{sum of roots})x\ +\ \text{product of roots}\right)$$
$$k\left\{x^2-\ 1x\ +\ 1\right\}$$
$$k\left\{x^2-\ x\ +\ 1\right\}$$

(v) $-\frac{1}{4}, \frac{1}{4}$

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2 - (\text{sum of roots})x + \text{product of roots}\right)$$

$$K\left(x^2 - \left(-\frac{1}{4}x\right) + \frac{1}{4}\right)$$

$$K\left(x^2 + \frac{1}{4}x + \frac{1}{4}\right)$$

(vi) 4,1

We know that the general equation of a quadratic polynomial is:

$$K\left(x^2 - (\text{sum of roots})x + \text{product of roots}\right)$$

$$k\left\{x^2 - 4x + 1\right\}$$

Chapter - 2: Polynomials

Exercise 2.3 (Page 36 of Grade 10 NCERT Textbook)

Q.1 Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

Difficulty Level: Medium

What is the unknown?
The quotient and remainder of the given polynomials.

Reasoning:
You can solve this question by following the steps given below:

First, arrange the divisor as well as dividend individually in decreasing order of their degree of terms.

In case of division, we seek to find the quotient. To find the very first term of the quotient, divide the first term of the dividend by the highest degree term in the divisor.

Now write the quotient.

Multiply the divisor by the quotient obtained. Put the product underneath the dividend.

Subtract the product obtained as happens in case of a division operation.

Write the result obtained after drawing another bar to separate it from prior operations performed.

Bring down the remaining terms of the dividend.

Again, divide the dividend by the highest degree term of the remaining divisor.

Repeat the previous three steps on the interim quotient.

Solution:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\quad - 2x \\ & - \qquad\quad + \\ \hline & -3x^2 + 7x - 3 \\ & -3x^2 \qquad + 6 \\ & + \qquad\quad - \\ \hline & \qquad 7x - 9 \\ \hline \end{array}$$

Quotient $= x - 3$, Remainder $= 7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

$= x^4 + 0.x^3 - 3x^2 + 4x + 5$

$$\begin{array}{r|l} & x^2 + x - 3 \\ \hline x^2 + 1 - x & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\ & x^4 - x^3 + x^2 \\ & - \quad + \quad - \\ \hline & x^3 - 4x^2 + 4x + 5 \\ & x^3 - x^2 + x \\ & - \quad + \quad - \\ \hline & -3x^2 + 3x + 5 \\ & -3x^2 + 3x - 3 \\ & + \quad - \quad + \\ \hline & \qquad 8 \end{array}$$

Quotient $= x^2 + x - 3$, Remainder $= 8$

(iii) $p(x) = x^4 - 5x + 6, \qquad g(x) = 2 - x^2$

$$= x^4 + 0.x^2 - 5x + 6$$

$$
\begin{array}{r|l}
 & x^4 - 2x^2 \\
2 - x^2 & x^4 - (5x) + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 - 4 \\
 & - \quad + \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

Quotient $= -x^2 - 2$, Remainder $= -5x + 10$

Q2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Difficulty Level: Medium

What is the unknown?

Whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

Reasoning:

To solve this question, follow the same procedure given in question no 1, you have to observe only one thing -if the remainder is 0 then the first polynomial is a factor of the second polynomial.

Solution:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$t^2 - 3 = t^2 + 0.t - 3$$

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4 \qquad -6t^2 \\
 & - \qquad + \\
\hline
 & \quad 3t^3+4t^2-9t-12 \\
 & \quad 3t^3 \qquad -9t \\
 & \quad - \qquad + \\
\hline
 & \quad 4t^2-12 \\
 & \quad 4t^2-12 \\
 & \quad - \qquad + \\
\hline
 & \qquad 0
\end{array}
$$

Since, the remainder is zero, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & - \qquad + \\
\hline
 & \quad -4x^3-10x^2+2x+2 \\
 & \quad -4x^3-12x^2-4x \\
 & \quad + \qquad + \qquad + \\
\hline
 & \qquad 2x^2+6x+2 \\
 & \qquad 2x^2+6x+2 \\
 & \qquad - \quad - \quad - \\
\hline
 & \qquad 0
\end{array}
$$

Quotient $=3x^2-4x+2$, Remainder $=0$

Since, the remainder is zero, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -x^3 + 3x + 1 \\ & -x^3 + 3x - 1 \\ & + \quad - \quad + \\ \hline & 2 \end{array}$$

Quotient $= x^2 - 1$, Remainder $= 2$

Since, the remainder is not zero,

$x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$

Q.3 Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Difficulty Level: Hard

What is given /known?

Two zeroes of a polynomial are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

What is the unknown?

All other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Reasoning:

You can solve this question by following the steps given below:

Given polynomial p(x) = $3x^4 + 6x^3 - 2x^2 - 10x - 5$,

Two zeroes of the polynomial are given as $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Therefore,

$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Divide $3x^4+6x^3-2x^2-10x-5$ by $\left(x^2-\frac{5}{3}\right)$

On dividing you will get the quotient and the remainder.

Put the values of dividend, divisor, quotient and remainder in the division algorithm.

Now you can easily find out the other zeroes of the polynomial.

Solution:

$$P(x) = 3x^4+6x^3-2x^2-10x-5$$

Since, the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Therefore, $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$\begin{array}{r|l} & 3x^2+6x+3 \\ \hline x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & 3x^4+0.x^3-5x^2 \\ & -\qquad + \\ \hline & 6x^3+3x^2-10x-5 \\ & 6x^3+0.x^2-10x \\ & -\qquad + \\ \hline & 3x^2+0x-5 \\ & 3x^2+0x-5 \\ & -\qquad + \\ \hline & 0 \end{array}$$

$$\therefore 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

$$= \left(x^2 - \frac{5}{3}\right)\left(3x^2 + 6x + 3\right) + 0$$

$$= 3\left(x^2 - \frac{5}{3}\right)\left(x^2 + 2x + 1\right)$$

We factorize $x^2 + 2x + 1 = (x+1)^2$
Therefore, its zero is given by $x + 1 = 0, x = -1$
As it has the term $(x+1)^2$.
Therefore, there will be two identical zeroes at $x = -1$.
Hence the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

Q.4 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Difficulty Level: Easy

What is given /known?
$P(x) = x^3 - 3x^2 + x + 2$

What is the unknown?
Divisor *g(x)* of a polynomial *p(x)*.

Reasoning:
This question is straight forward, you can solve it by using division algorithm:

Dividend = Divisor x Quotient + Remainder

Put the given values in the above equation and simplify it, get the value of *g(x)*.

Solution:

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$\left(x^3 - 3x^2 + x + 2\right) - \left(-2x + 4\right) = g(x) \times (x - 2)$$

$$\left(x^3 - 3x^2 + x + 2x + 2 - 4\right) = g(x) \times x - 2$$

$$\left(x^3 - 3x^2 + 3x - 2\right) = g(x) \times (x - 2)$$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Therefore, $g(x) = x^2 - x + 1$

Q.5 Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$ (ii) $\deg q(x) = deg\, r(x)$ (iii) $\deg r(x) = 0$

Difficulty Level: Easy

What is given /known?

(i) $\deg p(x) = deg\, q(x)$ (ii) $\deg q(x) = \deg r(x)$ (iii) $\deg r(x) = 0$

What is the unknown?

Examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm

Reasoning:

To solve this question, follow some steps:

In case (i), assume polynomial *p(x)* whose degree is equal to degree of *q(x)*, then put the values of *p(x)*, *g(x)*, *q(x)* and *r(x)*

In the division algorithm, if L.H.S is equal to R.H.S, then the division algorithm is satisfied.

In case (ii), assume polynomial *p(x)* in which degree of quotient *q(x)* is equal to the degree of *r(x)*, then put the values of *p(x)*, *g(x)*, *q(x)* and *r(x)* in the division algorithm. If L.H.S is equal to R.H.S then the division algorithm is satisfied.

In case (iii), assume polynomial *p(x)* in which degree of remainder *r(x)* is equal to zero, then put the values of *p(x)*, *g(x)*, *q(x)* and *r(x)* in the division algorithm.
If L.H.S is equal to R.H.S, then the division algorithm is satisfied.

Use the below given statement of Division algorithm to solve this question-

Division algorithm

Dividend = Divisor × Quotient + Remainder

According to division algorithm, if *p(x)* and *g(x)* are two polynomials with $g(x) \neq 0$, then we can find polynomial *q(x)* and *r(x)* such that

$$p(x) = g(x) \times q(x) + r(x)$$

Where $r(x) = 0$ or degree of *r(x)* < degree of *g(x)*

Degree of polynomial is the highest power of the variable in the polynomial.

Put the given values in the above equation and simplify it, get the value of *g(x)*.

Solution:

(i) $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to the degree of dividend when divisor is constant (i.e. when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2

$$p(x) = 6x^2 + 2x + 2$$
$$g(x) = 2$$
$$q(x) = 3x^2 + x + 1, \quad r(x) = 0$$

Degree of *p(x)* and *q(x)* is same i.e. 2.

Checking for division algorithm:

$$\begin{aligned} p(x) &= g(x) \times q(x) + r(x) \\ 6x^2 + 2x + 2 &= 2(3x^2 + x + 1) + 0 \\ &= 6x^2 + 2x + 2 \end{aligned}$$

Thus, the division algorithm is satisfied.

(ii) $\deg q(x) = \deg r(x)$

Let us assume the division of $x^3 + x$ by x^2

$$P(x) = x^3 + x$$
$$g(x) = x^2$$
$$q(x) = x, r(x) = x$$

Clearly, degree of *r(x)* and *q(x)* is same i.e. 1.

Checking for division algorithm

$$p(x) = g(x) \times q(x) + r(x)$$
$$x^3 + x = (x^2 \times x) + x$$
$$x^3 + x = x^3 + x$$

Thus, the division algorithm is satisfied.

(iii) $\deg r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2.

$P(x) = x^3 + 1$, $g(x) = x^2$, $q(x) = x$ and $r(x) = 1$

Clearly, the degree of *r(x)* is 0.

Checking for division algorithm

$$P(x) = g(x) \times q(x) + r(x)$$
$$x^3 + 1 = (x^2 \times x) + 1$$
$$x^3 + 1 = x^3 + 1$$

Thus, the division algorithm is satisfied.

Chapter - 2: Polynomials

Exercise 2.4 (Page 36 of Grade 10 NCERT Textbook)

Q1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$ (ii) $x^3 - 4x^2 + 5x - 2;\ 2, 1, 1$

Difficulty Level: Easy

What is given /known?

Cubic polynomials and their zeroes are given

What is the unknown?

Verify that the numbers given alongside of the cubic polynomials are their zeroes and verify the relationship between the zeroes and the coefficients in each case.

Reasoning:

This question is very simple, cubic polynomials are already given. Put the values of zeroes in the polynomial, you will get zero because it will satisfy the equation of the given polynomial. Now compare the polynomial with $ax^3 + bx^2 + cx + d$, you will get the values of the coefficients of x i.e. a, b, c and d.

Solution:

$p(x) = 2x^3 + x^2 - 5x + 2$

Given numbers are $\frac{1}{2}, 1, -2$

Put $x = \frac{1}{2}$ in $p(x) = 2x^3 + x^2 - 5x + 2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right) - 5\left(\frac{1}{2}\right) + 2$$

$$p\left(\frac{1}{2}\right) = \frac{1}{4} + \left(\frac{1}{4}\right) - \frac{5}{2} + 2$$

$$p\left(\frac{1}{2}\right) = \frac{1 + 1 - 10 + 8}{4}$$

$$p\left(\frac{1}{2}\right) = 0$$

Put $x = 1$ in $p(x) = 2x^3 + x^2 - 5x + 2$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$p\left(\frac{1}{2}\right) = 2 + 1 - 5 + 2$$

$$p(1) = 3 - 5 + 2$$

$$p(1) = 0$$

Put $x = -2$ in $p(x) = 2x^3 + x^2 - 5x + 2$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$p(-2) = -16 - 8 + 4 + 10 + 2$$

$$p(-2) = -16 + 4 + 10 + 2$$

$$p(-2) = -16 + 16$$

$$p(-2) = 0$$

Therefore, $\frac{1}{2}, 1, -2$ are the zeroes of the polynomial.

Now let $\alpha = \frac{1}{2}, \beta = 1$ and $\gamma = 2.$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2)$$

$$= \frac{-1}{2}$$

$$= \frac{-coeficient\ of\ x^2}{coeficient\ of\ x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2}$$

$$= \frac{-5}{2}$$

$$= \frac{-coeficient\ of\ x}{coeficient\ of\ x^3}$$

$$\alpha.\beta.\gamma = \frac{1}{2} \times 1 \times (-2)$$

$$= \frac{-2}{2}$$

$$= \frac{-\ constant\ term}{coeficient\ of\ x^3}$$

Hence, the relation between zeroes and coefficient is verified.

(ii) $x^3 - 4x^2 + 5x - 2.$

Given numbers are $2, 1, 1$

Put $x = 2$ in $x^3 - 4x^2 + 5x - 2$

$$p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$$
$$p(2) = 8 - 16 + 10 - 2$$
$$p(2) = 2 - 2$$
$$p(2) = 0$$

Put $x = 1$ in $x^3 - 4x^2 + 5x - 2$

$$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$$
$$p(1) = 1 - 4 + 5 - 2$$
$$p(1) = -3 + 3$$
$$p(1) = 0$$

Therefore, 2,1 and 1 are the zeroes of the polynomial.

Now let $\alpha = 2$, $\beta = 1$ and $\gamma = 1$.

$$\alpha + \beta + \gamma = 2 + 1 + 1$$
$$= \frac{-(-1)}{1}$$
$$= \frac{-\text{coeficient of } x^2}{\text{coeficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 5$$
$$= \frac{5}{1}$$
$$= \frac{-\text{coeficient of } x}{\text{coeficient of } x^3}$$

$$\alpha . \beta . \gamma = 2 \times 1 \times 1 = 2$$
$$= \frac{-(-2)}{1}$$
$$= \frac{-constant\ term}{coeficient\ of\ x^3}$$

Hence, the relation between zeroes and coefficient is verified.

Q2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Difficulty Level: Medium

What is given /known?

Zeroes of a Cubic polynomials are 2, –7, –14 respectively. What is the unknown?

What is unknown?

A cubic polynomial with the sum, sum of the product of its zeroes taken two at a time.

Reasoning:

To solve this question, follow the steps below- We know that the general equation of the polynomial is $ax^3 + bx^2 + cx + d$ and the zeroes are α, β and γ. Now we have to find the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes. For this we know that,

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Put the values of the known coefficients, you will get the value of unknown coefficient. Now put the values of coefficients in the general equation of the cubic polynomial $ax^3 + bx^2 + cx + d$.

Solution:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes are α, β and γ.

We know that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha.\beta \cdot \gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$ then $b = -2$, $c = -7$ and $d = 14$

Hence the polynomial is $x^3 - 2x^2 - 7x + 14$

Q3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a, $a + b$, find a and b.

Difficulty Level: Medium

What is given /known?

Zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a, $a + b$.

What is the unknown?

a and b.

Reasoning:

First compare the given polynomial with the general equation of the cubic polynomial $px^3 + qx^2 + rx + t$ and you will get the values of p, q, r and t. Now put this value in the equation of sum of zeroes and product of zeroes, you will get the value of a and b.

Solution:

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are $a - b,\ a,\ a + b.$

On comparing the given polynomial with $px^3 + qx^2 + rx + t$

we get, $p = 1,\ q = -3,\ r = 1$ and $t = 1$

sum of zeroes $= a - b + a + a + b$

$$= \frac{-coeficient\ of\ x^2}{coeficient\ of\ x^3} = \frac{-q}{p}$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$a = 1$$

Since the value of a is found to be 1, the zeroes are $1 - b,\ 1\ ,\ 1 + b$

$$\text{Multiplication of zeroes} = 1(1-b)(1+b) = \frac{-constant\ term}{coeficient\ of\ x^3} = \frac{-t}{p}$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b^2 = 2$$

$b = \pm\sqrt{2}.$ Hence, $a = 1,\ b = \sqrt{2}$ or $-\sqrt{2}$

Q4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ find other zeroes.

Difficulty Level: Medium

What is given /known?

Two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$.

What is the unknown?

Other zeroes of the given polynomial.

Reasoning:

Given polynomial and the zeroes of the polynomial are $x^4 - 6x^3 - 26x^2 + 138x - 35$ and $2 \pm \sqrt{3}$.

By using the zeroes of a polynomial, you can find out the factor of the polynomial.

Now divide the polynomial with the factor, you will get the quotient and remainder of the polynomial.

Put this value in the division algorithm and you will get the other zeroes by simplifying its factors.

Solution:

$P(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Zeroes of the polynomial are $= 2 \pm \sqrt{3}$.

Therefore, $\left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$$= x^2 - 4x + 1$$

is a factor of the given polynomial.

To find out the other polynomial, we have to find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Clearly, by division algorithm,

$$x^4 - 6x^3 - 26x^2 + 138x - 35 = \left(x^2 - 4x + 1\right)\left(x^2 - 2x - 35\right)$$

It can be observed that $x^2 - 2x - 35$ is a factor of the given polynomial and

$$x^2 - 2x - 35 = x^2 - 7x + 2x - 35$$

$$= (x - 7)(x - 5)$$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ and $x = -5$

Hence, 7 and -5 are also zeroes of this polynomial.

Q5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Difficulty Level: Medium

What is given /known?

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$.

What is the unknown?

Value of k and a.

Reasoning:

This question is straight forward, use division algorithm and put the known values of dividend, divisor and remainder in the division algorithm and get the Quotient. Now you will get the remainder, but we have already subtracted remainder that means ideally the remainder should be equal to zero. Now, on equating remainder with zero, you will get the value of k and a.

Solution:

By Division algorithm,

$$\text{Dividend} = \text{Divisior} \times \text{Quotient} + \text{Remainder}$$

$$x^4 - 6x^3 + 16x^2 - 25x + 10 = \left(x^2 - 2x + k\right) \times Q + \left(x + \mathrm{a}\right)$$

$$x^4 - 6x^3 + 16x^2 - 26x + 10 - a = \left(x^2 - 2x + k\right) \times Q$$

$$\frac{x^4 - 6x^3 - 26x + 10 - a}{\left(x^2 - 2x + k\right)} = Q$$

$$\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & \quad + \quad - \\
\hline
 & -4x^3 + (16 - k)x^2 - 26x + 10 - a \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26 + 4k)x + 10 - a \\
 & (8-k)x^2 - (16 + 2k)x + \left(8k - k^2\right) \\
 & \quad + \qquad\quad - \\
\hline
 & (-10 + 2k)x^2 + \left(10 - a - 8k + k^2\right)
\end{array}$$

On comparing

$$(-10 + 2k) = 0$$
$$2k = 10$$
$$k = 5$$

$$10 - a - 8k + k^2 = 0$$
$$10 - a - 8(5) + (5)^2 = 0$$
$$10 - a - 40 + 25 = 0$$
$$-30 + 25 - a = 0$$
$$-5 - a = 0$$
$$a = -5$$

Hence $k = 5,\ a = -5$