Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.1 (Page 44 of Grade 10 NCERT)

Q1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Difficulty Level: Medium

Known: 'a' can be $6q$ or $6q+1$ or $6q+2$ or $6q+3$, or $6q+4$ or $6q+5$.
7 years ago, Aftab was 7 times as old as his daughter then and 3 years from now, Aftab shall be 3 times as old as his daughter will be.

Unknown:
Represent the situation algebraically and graphically.

Reasoning:
Assume the present age of Aftab be x years and his daughter be y years then represent their ages 7 years later and 3 years ago in term of x and y. Two linear equations can be formed to represent the above situation algebraically.
Using algebraic equation and truth table they can be graphically represented.

Solution:

(i) Present age of Aftab $= x$ years and his daughter $= y$ years

Therefore, 7 years ago, age of Aftab $= (x-7)$ years and his daughter $= (y-7)$ years

Using this information and applying the known condition that 7 years ago, Aftab was 7 times as old as his daughter then:

$$x-7 = 7(y-7)$$
$$x-7 = 7y-49$$
$$x-7y-7+49 = 0$$
$$x-7y+42 = 0$$

After 3 years from now, age of Aftab $= (x+3)$ years and his daughter $= (y+3)$ years and also Aftab will be 3 times as old as his daughter. Then mathematically,

$$x+3=3(y+3)$$
$$x+3=3y+9$$
$$x-3y+3-9=0$$
$$x-3y-6=0$$

Algebraic representations, where *x* and *y* are present ages of Aftab and his daughter respectively:

$$x-7y+42=0 \qquad (1)$$
$$x-3y-6=0 \qquad (2)$$

Therefore, the algebraic representation is for equation 1 is:

$$x-7y+42=0$$
$$-7y=-x-42$$
$$7y=x+42$$
$$y=\frac{x+42}{7}$$

And, algebraic representation for equation (2) is:

$$x-3y-6=0$$
$$-3y=-x+6$$
$$3y=x-6$$
$$y=\frac{x-6}{3}$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

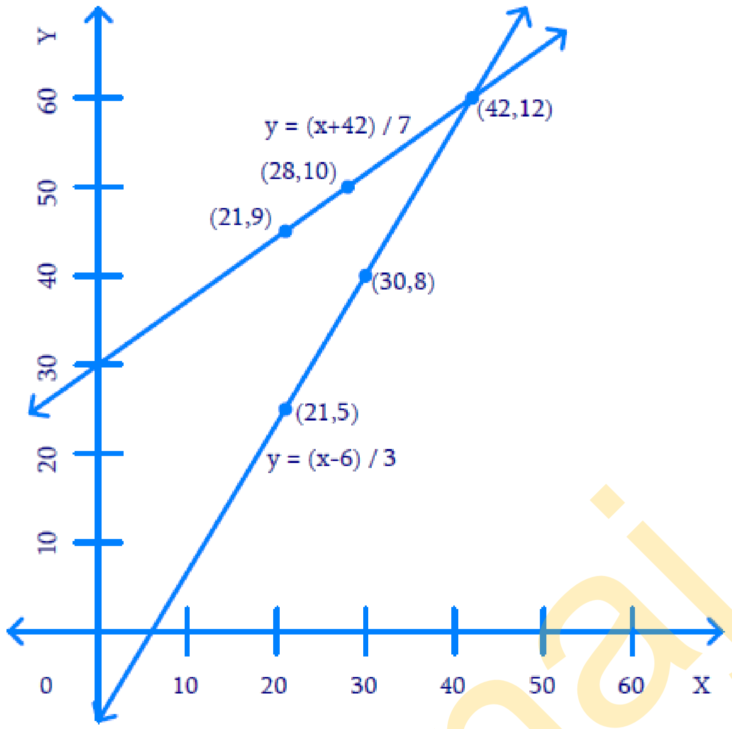
For equation (1)

x	21	28
$y=\frac{x+42}{7}$	9	10

For equation (2)

x	30	21
$y=\frac{x-6}{3}$	8	5

The graphical representation is as follows.



Unit: 1 cm = 5 years.

Answer: (42, 12)

Q2. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.

Difficulty Level: Easy

Known:

(i) Three bats and six balls for ₹ 3900
(ii) One bat and three balls for ₹ 1300

Unknown:
Represent the situation geometrically and algebraically

Reasoning:
Assuming the cost of one bat as ₹ x and the cost of one ball as ₹ y, two linear equations can be formed for the above situation.

Solution:

The cost of 3 bats and 6 balls is ₹ 3900. Mathematically:

$$3x+6y=3900$$
$$3(x+2y)=3900$$
$$x+2y=1300$$

Also, the cost of 1 bat and 3 balls is ₹ 1300. Mathematically:

$$x+3y=1300$$

Algebraic representation where x and y are cost of bat and ball respectively.

$$x+2y=1300 \qquad (1)$$
$$x+3y=1300 \qquad (2)$$

Therefore, the algebraic representation for equation 1 is:

$$x+2y=1300$$
$$2y=1300-x$$
$$y=\frac{1300-x}{2}$$

And, the algebraic representation for equation 2 is:

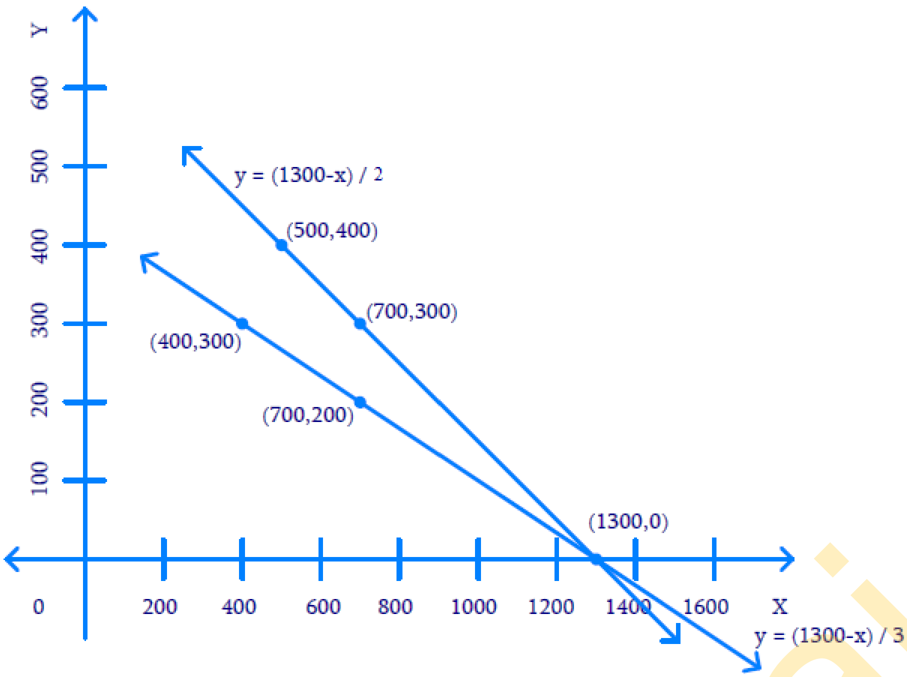
$$x+3y=1300$$
$$3y=1300-x$$
$$y=\frac{1300-x}{3}$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

x	700	500
$y=\frac{1300-x}{2}$	300	400

x	400	700
$y=\frac{1300-x}{3}$	300	200

The graphical representation is as follows.



Unit: 1cm = ₹ 100.

Answer: (1300, 0)

Q3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.

Difficulty Level: Easy

Known:

(i) Cost of 2 kg of apples and 1 kg of grapes is ₹ 160
(ii) Cost of 4 kg of apples and 2 kg of grapes is ₹ 300.

Unknown:
Represent the situation geometrically and algebraically.

Reasoning:
Assuming the cost of 1 kg apples as ₹ x and the cost of 1 kg grapes as ₹ y, two linear equations can be formed for the above situation.

Solution:

Let the cost of 1 kg of apples be x and cost of 1 kg of grapes be y

Cost kg 2 kg apples and 1 kg of grapes is ₹ 160. Mathematically,

$$2x + y = 160$$

Also, cost kg 4 kg apples and 2 kg of grapes is ₹ 300. Mathematically,

$$4x+2y=300$$
$$2(2x+y)=300$$
$$2x+y=150$$

Algebraic representation where x and y are the cost of 1 kg apple and 1 kg grapes respectively.

$$2x+y=160 \qquad (1)$$
$$2x+y=150 \qquad (2)$$

Therefore, the algebraic representation is for equation 1 is:

$$2x+y=160$$
$$y=160-2x$$

And, the algebraic representation is for equation 2 is:

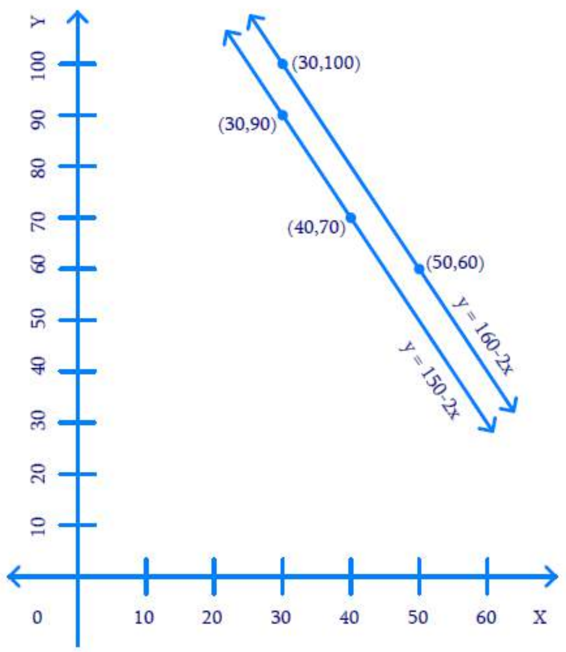
$$2x+y=150$$
$$y=150-2x$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

x	50	30
$y=160-2x$	60	100

x	30	40
$y=150-2x$	90	70

The graphical representation is as follows.



Unit = 1cm = ₹ 10

Answer:

Since the lines are parallel hence no Solution

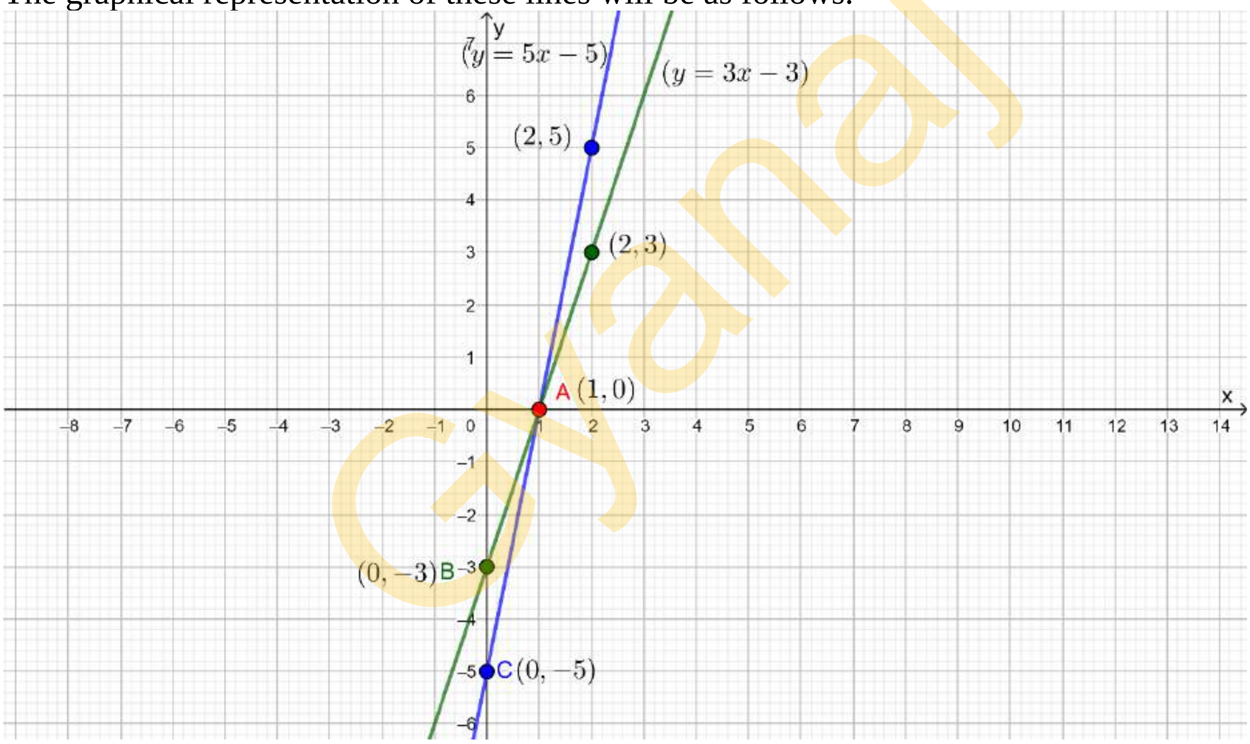
x	0	2
y	-5	5

$$3x - y = 3$$
$$\Rightarrow y = 3x - 3$$

The solution table will be as follows.

x	0	2
y	-3	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ABC formed by these lines and y-axis.

The coordinates of vertices are A $(1, 0)$, B $(0, -3)$, C $(0, -5)$.

Q7. Solve the following pair of linear equations.

(i) $px + qy = p - q$
$qx - py = p + q$

(ii) $ax + by = c$
$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$
$ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$
$(a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74$
$-378x + 152y = -604$

Difficulty Level: Medium

Solution:

$(i) \quad px + qy = p - q \qquad \ldots(1)$

$qx - py = p + q \qquad \ldots(2)$

Multiplying equation (1) by p and equation (2) by q, we obtain

$$p^2x + pqy = p^2 - pq \qquad \ldots(3)$$

$$q^2x - pqy = pq + q^2 \qquad \ldots(4)$$

Adding equations (3) and (4), we obtain

$$p^2x + q^2x = p^2 + q^2$$

$$\left(p^2 + q^2\right)x = p^2 + q^2$$

$$x = \frac{p^2 + q^2}{p^2 + q^2}$$

$$x = 1$$

Substituting $x = 1$ in equation (1), we obtain

$$p \times 1 + qy = p - q$$

$$qy = -q$$

$$y = -1$$

Therefore, $x = 1$ and $y = -1$

$(ii) \quad ax + by = c \qquad \ldots(1)$

$bx + ay = 1 + c \qquad \ldots(2)$

Multiplying equation (1) by a and equation (2) by b, we obtain

$$a^2x + aby = ac \qquad \ldots(3)$$

$$b^2x + aby = b + bc \qquad \ldots(4)$$

Subtracting equation (4) from equation (3),

$$\left(a^2 - b^2\right)x = ac - bc - b$$

$$x = \frac{c(a-b) - b}{a^2 - b^2}$$

Substituting $x=\frac{c(a-b)-b}{a^2-b^2}$ in equation (1), we obtain

$$ax+by = c$$

$$a\left(\frac{c(a-b)-b}{a^2-b^2}\right)+by = c$$

$$\frac{ac(a-b)-ab}{a^2-b^2}+by = c$$

$$by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$by=\frac{bc(a-b)+ab}{a^2-b^2}$$

$$by=\frac{b\left[c(a-b)+a\right]}{a^2-b^2}$$

$$y=\frac{c(a-b)+a}{a^2-b^2}$$

Therefore, $x=\frac{c(a-b)-b}{a^2-b^2}$ and $y=\frac{c(a-b)+a}{a^2-b^2}$

(iii) $\frac{x}{a}-\frac{y}{b}=0$ $\qquad\ldots(1)$

$ax+by=a^2+b^2$ $\qquad\ldots(2)$

By solving equation (1), we obtain

$$\frac{x}{a}-\frac{y}{b}=0$$

$$x=\frac{ay} {b}\qquad\ldots(3)$$

Substituting $x=\frac{ay}{b}$ in equation (2), we obtain

$$a\times\left(\frac{ay}{b}\right)+by=a^2+b^2$$

$$\frac{a^2y+b^2y}{b}=a^2+b^2$$

$$\left(a^2+b^2\right)y=b\left(a^2+b^2\right)$$

$$y=b$$

Substituting $y=b$ in equation (3), we obtain

$$x=\frac{a\times b}{b}$$

$$x=a$$

Therefore, $x=a$ and $y=b$

(iv) $(a-b)x+(a+b)y=a^2-2ab-b^2$ $\quad\ldots(1)$

$(a+b)(x+y)=a^2+b^2$ $\quad\ldots(2)$

By solving equation (2), we obtain

$$(a+b)(x+y)=a^2+b^2$$

$$(a+b)x+(a+b)y=a^2+b^2 \quad\ldots(3)$$

Subtracting equation (3) from (1), we obtain

$$(a-b)x-(a+b)x=\left(a^2-2ab-b^2\right)-\left(a^2+b^2\right)$$

$$\left[(a-b)-(a+b)\right]x=a^2-2ab-b^2-a^2-b^2$$

$$\left[a-b-a-b\right]x=-2ab-2b^2$$

$$-2bx=-2b\left(a+b\right)$$

$$x=\left(a+b\right)$$

Substituting $x=\left(a+b\right)$ in equation (1), we obtain

$$(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$$

$$(a^2-b^2)+(a+b)y=a^2-2ab-b^2$$

$$(a+b)y=a^2-2ab-b^2-(a^2-b^2)$$

$$(a+b)y=a^2-2ab-b^2-a^2+b^2$$

$$y=\frac{-2ab}{(a+b)}$$

(v) $152x-378y=-74$ $\quad\ldots(1)$

$-378x+152y=-604$ $\quad\ldots(2)$

Adding equations (1) and (2), we obtain

$$-226x-226y=-678$$

$$-226\left(x+y\right)=-678$$

$$x+y=3 \quad\ldots(3)$$

Subtracting equation (2) from (1), we obtain

$$530x-530y=530$$

$$530\left(x-y\right)=530$$

$$x-y=1 \quad\ldots(4)$$

Adding equations (3) and (4), we obtain

$$2x=4$$

$$x=2$$

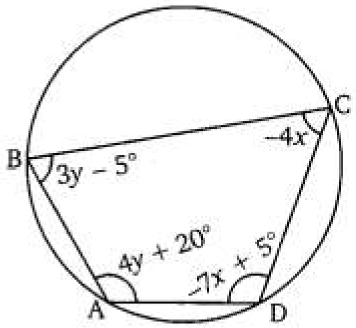
Substituting $x=2$ in equation (3), we obtain

$$2+y=3$$

$$y=1$$

Therefore, $x=2$ and $y=1$

Q8. ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Difficulty Level: Medium

Known:
Measurement of the angles of the cyclic quadrilateral in terms of x and y.
Unknown:
Measurement of the angles of the cyclic quadrilateral.
Reasoning:
Pairs of opposite angles of a cyclic quadrilateral are supplementary.
Solution:
We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.
Therefore,

$$\angle A+\angle C=180^{\circ}$$
$$(4y+20)+(-4x)=180$$
$$4y+20-4x=180$$
$$-4(x-y)=160$$
$$x-y=-40 \qquad (1)$$

And

$$\angle B+\angle D=180^{\circ}$$
$$(3y-5)+(-7x+5)=180$$
$$3y-5-7x+5=180$$
$$-7x+3y=180$$
$$7x-3y=-180 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$$3x-3y=-120 \qquad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$4x=-60$$
$$x=-15$$

Substituting $x=-15$ in equation (1), we obtain

$$-15-y=-40$$
$$y=25$$

Therefore,

$$\angle A = 4\times 25 + 20 = 120^{\circ}$$

$$\angle B = 3\times 25 - 5 = 70^{\circ}$$

$$\angle C = -4\times(-15) = 60^{\circ}$$

$$\angle D = -7\times(-15) + 5 = 110^{\circ}$$

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.2 (Page 49 of Grade 10 NCERT)

Q1. Form the pair of linear equations in the following problems and find their Solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Difficulty Level: Medium

(i) Known:

(i) Number of students took part in Quiz = 10
(ii) Number of girls is 4 more than number of boys

Unknown:
Finding Solutions graphically for the given situation.

Reasoning:
Assuming the number of boys as *x* and the number of girls as *y*, two linear equations can be formed for the above situation.

Solution:
Total number of boys and girls is:

$$x+y=10$$

Number of girls is 4 more than the number of boys, Mathematically:

$$y=x+4$$
$$-x+y=4$$

Algebraic representation where *x* and *y* are the number of boys and girls respectively.

$$x+y=10 \qquad (1)$$
$$-x+y=4 \qquad (2)$$

Therefore, the algebraic representation for equation 1 is:

$$x+y=10$$
$$y=10-x$$

And, the algebraic representation is for equation 2 is:

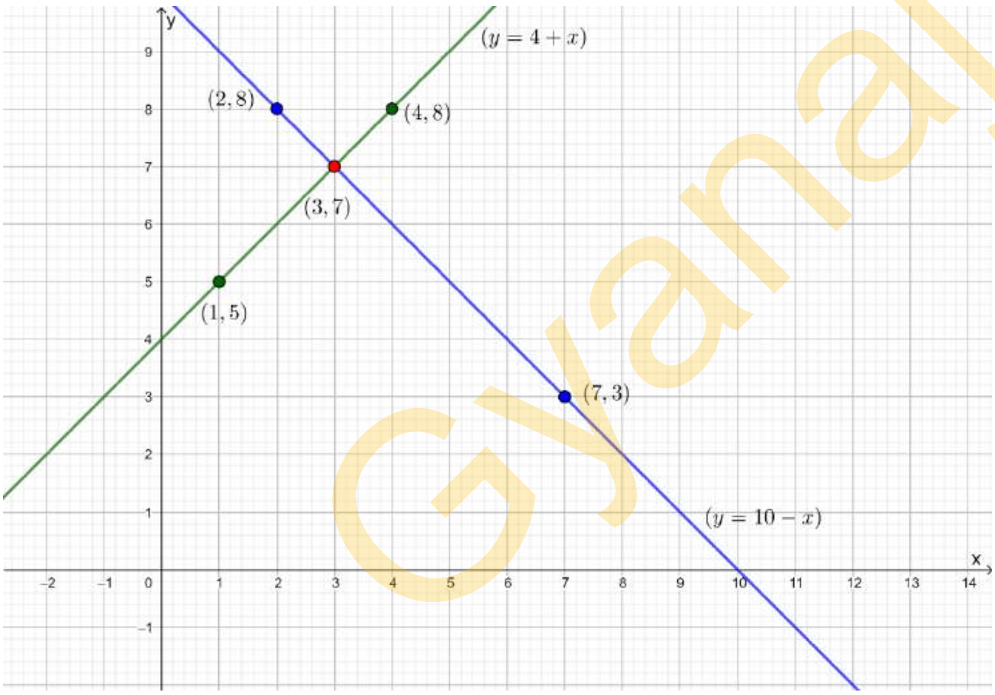
$$-x + y = 4$$
$$y = x + 4$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

x	2	7
$y = 10 - x$	8	3

x	1	4
$y = x + 4$	5	8

The graphical representation is as follows.



Answer:

From graph solution $(x, y) = (3, 7)$

Number of boys = 3

Number of girls = 7

(ii) Known:

(i) 5 pencils and 7 pens cost ₹ 50

(ii) 7 pencils and 5 pens cost ₹ 46

Unknown:

Finding Solutions graphically for the given situation.

Reasoning:

Assuming the cost of 1 pencil as ₹ x and the cost of 1 pen as ₹ y, two linear equations are to be formed for the above situation.

Solution:

Let us assume cost of 1 pencil be x and cost of 1 pen be y.

The cost of 5 pencils and 7 pens is ₹ 50. Mathematically,

$$5x+7y=50$$

And, the cost of 7 pencils and 5 pens is ₹ 50. Mathematically,

$$7x+5y=46$$

Algebraic representation where x and y are the cost of 1 pencil and 1 pen respectively.

$$5x+7y=50 \qquad (1)$$
$$7x+5y=46 \qquad (2)$$

Therefore, the algebraic representation for equation 1 is:

$$5x + 7y = 50$$
$$7y = 50 - 5x$$
$$y=\frac{50-5x}{7}$$

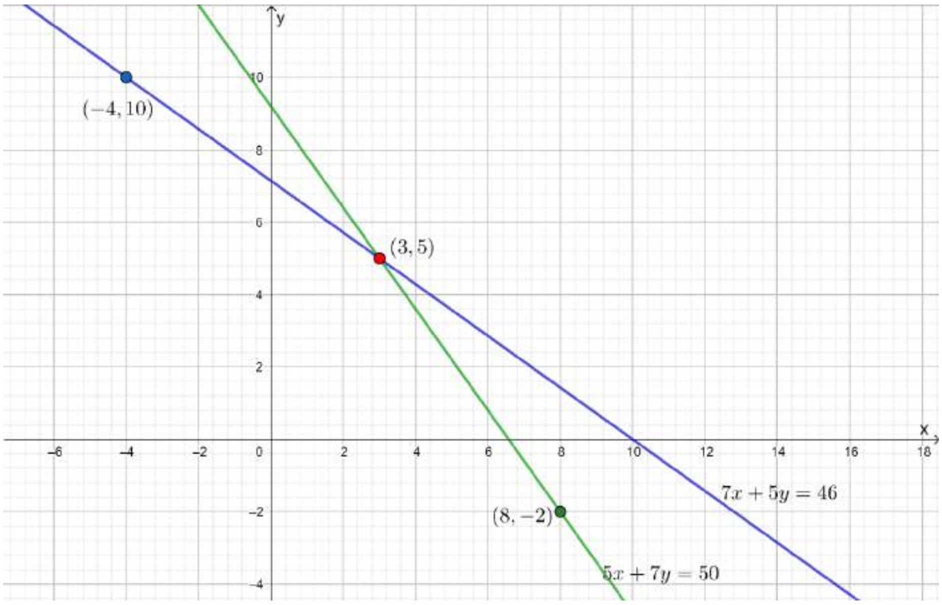
And, the algebraic representation for equation 2 is:

$$7x + 5y = 46$$
$$5y = 46 - 7x$$
$$y=\frac{46-7x}{5}$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

x	3	–4
$y=\frac{50-5x}{7}$	5	10

x	3	8
$y=\frac{46-7x}{5}$	5	–2



From graph Solution $(x, y) = (3, 5)$
Cost of one pencil = ₹ 3
Cost of one pen = ₹ 5

Q2. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

Difficulty Level: Easy

(i) Known:

$5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Unknown:

Whether the lines are
(i) Intersecting
(ii) Parallel
(iii) Coincident

Reasoning:

For any pair of linear equation

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$a)\ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\textbf{Intersecting Lines})$$

$$b)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\textbf{Coincident Lines})$$

$$c)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad (\textbf{Parallel Lines})$$

Solution:

$$a_1 = 5, \quad b_1 = -4 \quad c_1 = 8$$
$$a_2 = 7 \quad b_2 = 6 \quad c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{5}{7} \qquad ...(1)$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \qquad ...(2)$$

From (i) and (ii)

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they are intersecting lines at a point

(ii) Known:

$9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Unknown:

Whether the lines are

(i) Intersecting
(ii) Parallel
(iii) Coincident

Reasoning:

For any pair of linear equation

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$a)\ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\textbf{Intersecting Lines})$$

$$b)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\textbf{Coincident Lines})$$

$$c)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad (\textbf{Parallel Lines})$$

Solution:

$$a_1 = 9, \quad b_1 = 3 \quad c_1 = 12$$
$$a_2 = 18 \quad b_2 = 6 \quad c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2} \quad \text{...(1)}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \quad \text{...(2)}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2} \quad \text{...(3)}$$

From (1), (2) and (3)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Therefore, they are coincident lines

(iii) Known:

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Unknown:

Whether the lines

(i) Intersecting
(ii) Parallel
(iii) Coincident

Reasoning:

For any pair of linear equation

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$a)\ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\textbf{Intersecting Lines})$$

$$b)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\textbf{Coincident Lines})$$

$$c)\ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad (\textbf{Parallel Lines})$$

Solution:

$$a_1 = 6, \quad b_1 = -3 \quad c_1 = 10$$
$$a_2 = 2 \quad b_2 = -1 \quad c_2 = 9$$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3 \quad \text{...(1)}$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3 \quad \text{...(2)}$$

$$\frac{c_1}{c_2} = \frac{10}{9} \quad \text{...(3)}$$

From (1), (2) and (3)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, they are parallel lines.

Q3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5;\ 2x - 3y = 7$

(ii) $2x - 3y = 8;\ 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

(iv) $5x - 3y = 11;\ -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

Difficulty Level: Easy

Unknown:

To find out whether the linear equations are consistent or inconsistent.

Reasoning:

For any pair of linear equation

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Consistent means pair of linear equations have one solution or infinitely many solutions.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\textbf{Intersecting lines / one Solution})$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\textbf{Coincident Lines / Infinitely many Solutions})$$

Inconsistent means, the lines may be parallel and do not have any Solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad (\textbf{Parallel lines / No Solution})$$

(i) Known:

$$3x + 2y - 5 = 0$$
$$2x - 3y - 7 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{3}{2}$$

$$\frac{b_1}{b_2} = \frac{2}{-3}$$

$$\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

From above

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, lines are intersecting and have one solution,
Hence, the pair of equations are consistent.

(ii) Known:

$$2x - 3y - 8 = 0$$

$$4x - 6y - 9 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

From above

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, lines are parallel and have no solution,
Hence, the pair of equations are inconsistent.

(iii) Known:

$$\frac{3}{2}x + \frac{5}{3}y = 7$$

$$9x - 10y = 14$$

Solution:

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6}$$

$$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{3} \times \frac{1}{-10} = \frac{1}{-6}$$

$$\frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

From above

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, lines are intersecting and have one solution.
Hence, they are consistent.

(iv) Known:

$$5x - 3y - 11 = 0$$

$$-10x + 6y + 22 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$$

From above

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, lines are coincident and have infinitely many solutions.
Hence, they are consistent.

(v) Known:

$$\frac{4}{3}x + 2y = 8$$

$$2x + 3y = 12$$

Solution:

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

From above

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, lines are coincident and have infinitely many solutions.
Hence, they are consistent.

Q4. Which of the following pairs of linear equations are consistent / inconsistent? If consistent, obtain the Solution graphically:

(i) $x+y=5,\ 2x+2y=10$

(ii) $x-y=8,\ 3x-3y=16$

(iii) $2x+y-6=0,\ 4x-2y-4=0$

(iv) $2x-2y-2=0,\ 4x-4y-5=0$

Difficulty Level: Easy

Unknown:

Whether the linear equations are consistent or inconsistent and graphical solution, if consistent.

Reasoning:

Consistent means pair of linear equations have one solution or infinitely many solutions.

$$a_1x+b_1y+c_1=0$$

$$a_2x+b_2y+c_2=0$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2} \qquad (\textbf{Intersecting lines / one Solution})$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \qquad (\textbf{Coincident Lines / Infinitely many Solutions})$$

Inconsistent means, the lines may be parallel and do not have any Solution)

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \qquad (\textbf{Parallel lines / No Solution})$$

(i) Known:

$$x+y-5=0$$

$$2x+2y-10=0$$

Solution:

$$\frac{a_1}{a_2}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2}$$

From above

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, lines are coincident and have infinitely many solutions.

Hence, they are consistent.

$$x+y-5=0$$

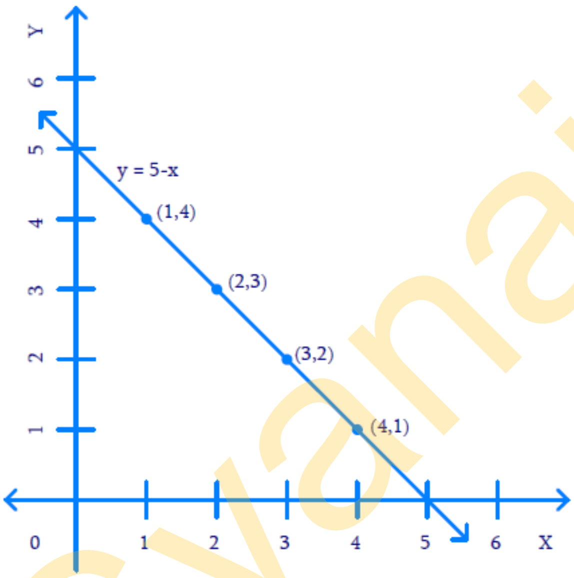
$$y=-x+5$$

$$y=5-x$$

x	1	2
$y = 5 - x$	4	3

$$2x + 2y - 10 = 0$$
$$2y = 10 - 2x$$
$$y = 5 - x$$

x	3	4
$y = 5 - x$	2	1



All the points on coincident line are solutions for the given pair of equations.

(ii) Known:

$$x - y - 8 = 0$$
$$3x - 3y - 16 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{1}{3}$$
$$\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$$
$$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

From above

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

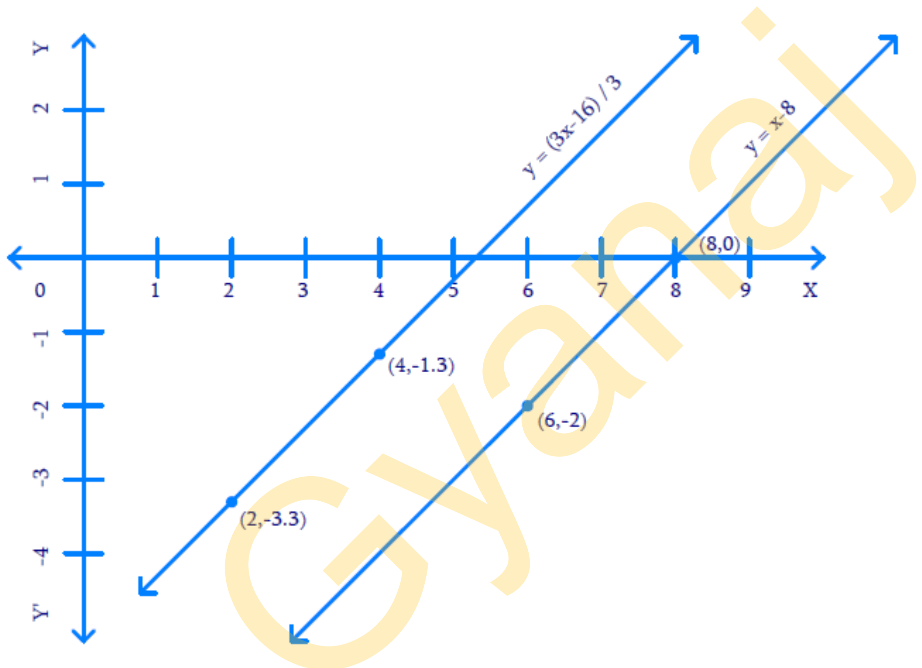
Therefore, lines are parallel and have no solution,
Hence, the pair of equations are inconsistent.

$$x - y - 8 = 0$$
$$y = x - 8$$

x	8	6
$y = x - 8$	0	-2

$$3x - 3y - 16 = 0$$
$$3y = 3x - 16$$
$$y = \frac{3x-16}{3}$$

x	2	4
$y = \frac{3x-16}{3}$	-3.3	-1.3



(iii) Known:

$$2x + y - 6 = 0$$
$$4x - 2y - 4 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2}$$
$$\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

From above:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, lines are intersecting and have one solution.

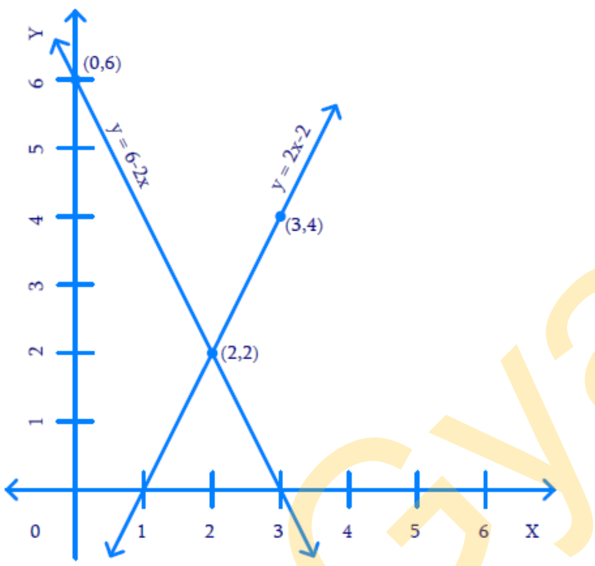
Hence, they are consistent.

$$2x + y - 6 = 0$$
$$y = 6 - 2x$$

x	0	2
$y = 6 - 2x$	6	2

$$4x - 2y - 4 = 0$$
$$2y = 4x - 4$$
$$y = 2x - 2$$

x	2	3
$y = 2x - 2$	2	4



$x = 2$ and $y = 2$ are solutions for the given pair of equations.

(iv) Known:

$$2x - 2y - 2 = 0$$
$$4x - 4y - 5 = 0$$

Solution:

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$$
$$\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

From above:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

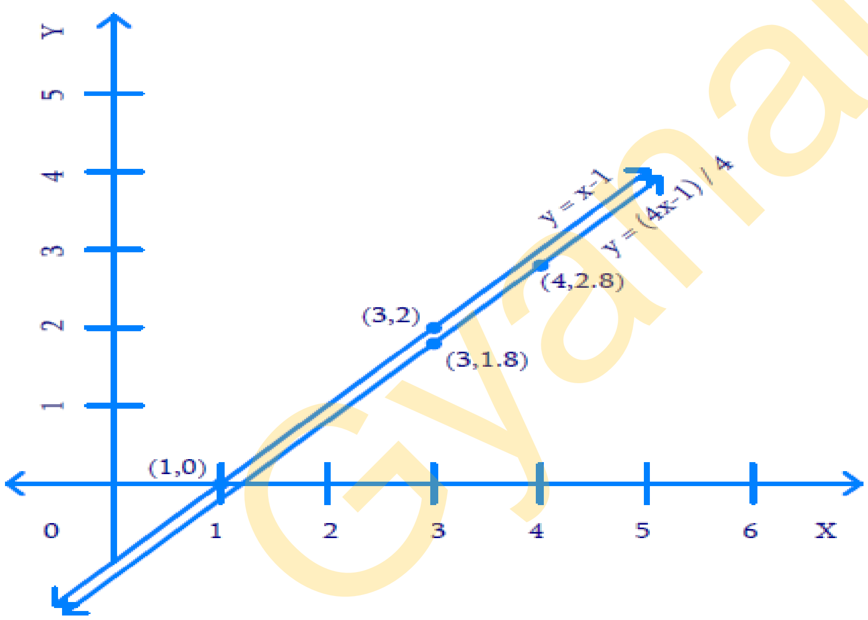
Therefore, lines are parallel and have no solution, Hence, the pair of equations are inconsistent.

$$2x-2y-2=0$$
$$2y=2x-2$$
$$y=x-1$$

x	1	3
$y=x-1$	0	2

$$4x-4y-5=0$$
$$4y=4x-5$$
$$y=\frac{4x-5}{4}$$

x	4	3
$y=\frac{4x-5}{4}$	2.8	1.8



Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Difficulty Level: Medium

Known:

(i) Half the perimeter of rectangular garden $=36\ m$
(ii) Length is $4\ m$ more than width

Unknown:
Dimensions of the garden

Reasoning:

Assuming length of the garden as x and width of the garden as y, two linear equations can be formed for the known data. Perimeter of rectangle = 2(length + breadth)

Solution:

Let the length of the garden be x and breadth be y

Then $x = y + 4$

$x - y = 4$

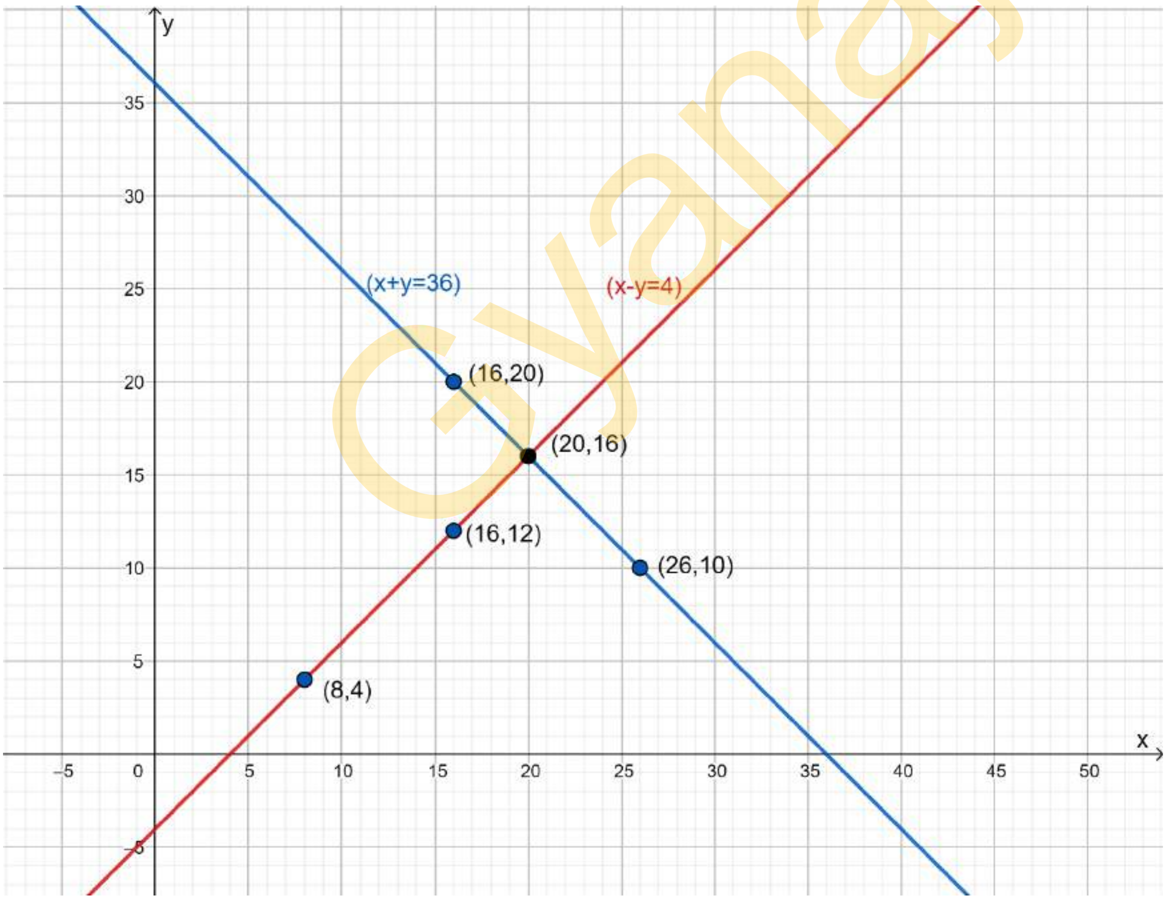
$y = x - 4$

x	8	16
$y = x - 4$	4	12

Half perimeter of the rectangle be $x + y = 36$

$y = 36 - x$

x	16	26
$y = x - 4$	20	10



Answer:

Length $x = 20\ m$

Breadth $y = 16\ m$

Q6. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i). intersecting lines
(ii). parallel lines
(iii). coincident lines

Difficulty Level: Medium

Known:

One linear equation $2x+3y-8=0$

Unknown:

Another linear equation such that given is satisfied.

Reasoning: **Same as Exercise 3.2 (2)**

(i) Intersecting lines

Condition: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$2x+3y-8=0$$
$$a_1 = 2$$
$$b_1 = 3$$

So, considering $a_2 = 3$ and $b_2 = 2$ will satisfy the condition for intersecting lines c_2 can be any value.

$$\frac{a_1}{a_2} = \frac{2}{3} \qquad \frac{b_1}{b_2} = \frac{3}{2}$$
$$\frac{2}{3} \neq \frac{3}{2}$$

$\therefore$ Another linear equation is $3x+2y-6=0$

(ii) Parallel Lines

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$2x+3y-8=0$$
$$a_1 = 2$$
$$b_1 = 3$$
$$c_1 = -8$$

So, considering $a_2 = 4,\ b_2 = 6,\ c_2 = 9$ will satisfy the condition for parallel lines.

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{c_1}{c_2} = \frac{-8}{9}$$

From above:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, another linear equation is $= 4x + 6y + 9 = 0$

(iii) Coincident lines:

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$2x + 3y - 8 = 0$$
$$a_1 = 2$$
$$b_1 = 3$$
$$c_1 = -8$$

So, considering $a_2 = 4,\ b_2 = 6,\ c_2 = -16$ will satisfy the condition for parallel lines.

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

From above:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, linear equation is $4x + 6y - 16 = 0$

Q7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis and shade the triangular region.

Difficulty Level: Medium

Known:

linear equation

$$x - y + 1 = 0$$
$$3x + 2y - 12 = 0$$

Unknown:
Coordinates of the vertices of the triangle formed by intersecting lines and the x-axis

Reasoning:
From graph of two linear equations and x-axis, triangle can be shaded, and vertices can be located.

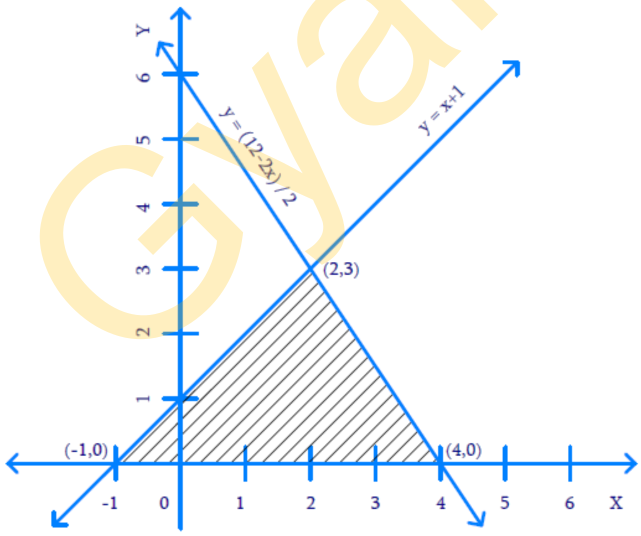
Solution:

$$x - y + 1 = 0$$
$$y = x + 1$$

x	0	1
$y = x + 1$	1	2

$$3x + 2y - 12 = 0$$
$$2y = 12 - 3x$$
$$y = \frac{12 - 3x}{2}$$

x	0	2
$y = \frac{12-3x}{2}$	6	3



From graph, Vertices are (-1, 0), (4, 0), and (2, 3)

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.3 (Page 53)

Q1. Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$, $x-y=4$

(ii) $s-t=3$, $\frac{s}{3}+\frac{t}{2}=6$

(iii) $3x-y=3$, $9x-3y=9$

(iv) $0.2x+0.3y=1.3$, $0.4x+0.5y=2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$, $\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$, $\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Difficulty Level: Easy

Known:
Pair of linear equations.

Unknown:
Solution for the given pair of linear equations.

Reasoning
Pick any one of two equations, write one variable in terms of other. Now substituting this in other equation will result in one variable equation and easy to solve.

(i) Solution

$$x+y=14 \quad \ldots(1)$$
$$x-y=4 \quad \ldots(2)$$

By solving the equation (1)

$$y=14-x \quad \ldots(3)$$

Substitute $y=14-x$ in equation (2), we get

$$x-(14-x)=4$$
$$2x-14=4$$
$$2x=4+14$$
$$2x=18$$
$$x=9$$

Substituting $x=9$ in equation (3), we get

$$y=14-9$$
$$y=5$$

Answer:

$$x=9$$
$$y=5$$

(ii) Solution

$$s-t=3 \qquad ...(1)$$
$$\frac{s}{3}+\frac{t}{2}=6 \qquad ...(2)$$

By solving the equation (1)

$$s-t=3$$
$$s=3+t \qquad ...(3)$$

Substitute $s=3+t$ in equation (2), we get

$$\frac{3+t}{3}+\frac{t}{2}=6$$
$$\frac{6+2t+3t}{6}=6$$
$$6+5t=6\times 6$$
$$5t=36-6$$
$$t=\frac{30}{5}$$
$$t=6$$

Substituting $t=6$ in equation (3), we get

$$s=3+6$$
$$s=9$$

Answer:

$$s=9$$
$$t=6$$

(iii) Solution

$$3x-y=3 \qquad ...(1)$$
$$9x-3y=9 \qquad ...(2)$$

By solving the equation (1)

$$3x-y=3$$
$$y=3x-3 \qquad ...(1)$$

Substitute $y=3x-3$ in equation (2), we get

$$9x-3(3x-3)=9$$
$$9x-9x+9=9$$
$$9=9$$

Shows that the lines are coincident and having infinitely many solutions.

Answer:

$$y=3x-3$$

Where x can take any value. i.e. Infinitely many Solutions.

(iv) Solution

$$0.2x+0.3y=1.3 \qquad ...(1)$$
$$0.4x+0.5y=2.3 \qquad ...(2)$$

Multiply both the equations (1) and (2) by 10, to remove the decimal number and making it easier for calculation.

$$[0.2x+0.3y=1.3]\times(10)$$
$$\Rightarrow 2x+3y=13 \qquad ...(3)$$

$$[0.4x+0.5y=23]\times(10)$$
$$\Rightarrow 4x+5y=23 \qquad ...(4)$$

By solving the equation (3)

$$2x+3y=13$$
$$3y=13-2x$$
$$y=\frac{13-2x}{3} \qquad ...(5)$$

Substitute $y=\frac{13-2x}{3}$ in equation (4), we get

$$4x+5\left(\frac{13-2x}{3}\right)=23$$
$$\frac{12x+65-10x}{3}=23$$
$$2x+65=23\times 3$$
$$2x=69-65$$
$$x=\frac{4}{2}$$
$$x=2$$

Substituting $x=2$ in equation (5), we get

$$y=\frac{13-2\times 2}{3}$$

$$y=\frac{9}{3}$$

$$y=3$$

Answer:

$$x=2$$

$$y=3$$

(v) Solution

$$\sqrt{2}x+\sqrt{3}y=0 \qquad ...(1)$$

$$\sqrt{3}x-\sqrt{8}y=0 \qquad ...(2)$$

By solving the equation (1)

$$\sqrt{2}x+\sqrt{3}y=0$$

$$\sqrt{3}y=-\sqrt{2}x$$

$$y=-\frac{\sqrt{2}x}{3} \qquad ...(3)$$

Substitute $y=-\frac{\sqrt{2}x}{3}$ in equation (2), we get

$$\sqrt{3}x-\sqrt{8}\left(\frac{-\sqrt{2}x}{3}\right)=0$$

$$\sqrt{3}x+\frac{\sqrt{16}x}{3}=0$$

$$\frac{3\sqrt{3}x+4x}{3}=0$$

$$x\left(3\sqrt{3}x+4\right)=0$$

$$x=0$$

Substituting $x=0$ in equation (3), we get

$$y=\frac{\sqrt{2}\times 0}{3}$$

$$y=0$$

Answer:

$$x=0$$

$$y=0$$

(vi) Solution

$$\frac{3x}{2}-\frac{5y}{3}=-2 \qquad ...(1)$$

$$\frac{x}{3}+\frac{y}{2}=\frac{13}{6} \qquad ...(2)$$

Multiply both the equations (1) and (2) by 6, to remove the decimal number and making it easier for calculation.

$$\left[\frac{3x}{2}-\frac{5y}{3}=-2\right]\times 6$$

$$9x-10y=-12 \qquad \text{...(3)}$$

$$\left[\frac{x}{3}+\frac{y}{2}=\frac{13}{6}\right]\times 6$$

$$2x+3y=13 \qquad \text{...(4)}$$

By solving the equation (3)

$$9x-10y=-12$$

$$10y=9x+12$$

$$y=\frac{9x+12}{10} \qquad \text{...(5)}$$

Substituting $y=\frac{9x+12}{10}$ in equation (4), we get

$$2x+3\left(\frac{9x+12}{10}\right)=13$$

$$\frac{20x+27x+36}{10}=13$$

$$47x=130-36$$

$$x=\frac{94}{47}$$

$$x=2$$

Substituting $x=2$ in equation (5), we get

$$y=\frac{9\times 2+12}{10}$$

$$y=\frac{30}{10}$$

$$y=3$$

Answer :

$$x=2$$

$$y=3$$

Q2. Solve $2x+3y=11$ and $2x-4y=-24$, hence find the value of 'm' for which $y=mx+3$.

Difficulty Level: Medium

Reasoning

Solve the linear equations (1) and (2) by substitution method and substitute the values of x and y in $y=mx+3$ to get the value of m.

Known:

$$2x+3y=11$$
$$2x-4y=-24$$
$$y=mx+3$$

Unknown:

Value of m

Solution

$$2x+3y=11 \qquad ...(1)$$
$$2x-4y=-24 \qquad ...(2)$$

By solving the equation (1)

$$2x+3y=11$$
$$3y=11-2x$$
$$y=\frac{11-2x}{3} \qquad ...(3)$$

Substituting $y=\dfrac{11-2x}{3}$ in equation (2), we get

$$2x-4\left(\frac{11-2x}{3}\right)=-24$$
$$\frac{6x-44+8x}{3}=-24$$
$$14x-44=-72$$
$$14x=44-72$$
$$x=-\frac{28}{14}$$
$$x=-2$$

Substituting $x=-2$ in equation (3)

$$y = \frac{11 - 2\times(-2)}{3}$$
$$y = \frac{11 + 4}{3}$$
$$y = \frac{15}{3}$$
$$y = 5$$

Now, Substituting $x = -2$ and $y = 5$ in $y = mx + 3$

$$y = mx + 3$$
$$5 = m(-2) + 3$$
$$5 - 3 = -2m$$
$$2 = -2m$$
$$m = \frac{2}{-2}$$
$$m = -1$$

Answer:

$$x = -2$$
$$y = 5$$
$$m = -1$$

Q3. Form the pair of linear equations for the following problems and find their Solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Difficulty Level: Medium

Unknown:
Formation of the pair of linear equations and their solution.

(i) Known:
The difference between two numbers is 26 and one number is three times the other.

Reasoning:

Assuming the numbers as x and y, two linear equations can be formed for the known situation.

Solution:
Let the first (larger) number $= x$
And the second number $= y$

The difference between two numbers is 26.

$$x - y = 26 \qquad \ldots(1)$$

One number is three times the other

$$x = 3y \qquad \ldots(2)$$

Substituting $x = 3y$ in equation (1), we get

$$3y - y = 26$$
$$2y = 26$$
$$y = 13$$

Substituting $y = 13$ in equation (2)

$$x = 3 \times 13$$
$$x = 39$$

Answer:
The two numbers are 39 and 13.

(ii) Known:
Larger of two supplementary angles, exceeds the smaller by 18 degrees.

Reasoning:
Supplementary angles are two angles with a sum of 180° and assuming the angles as x° and y°, two linear equations can be formed for the known situation.

Solution:
Let the larger angle $= x^\circ$
and smaller angle $= y^\circ$

Since the angles are supplementary
$$x^\circ + y^\circ = 180 \qquad \ldots(1)$$

Larger angle exceeds the smaller by 18°
$$x^\circ = y^\circ + 18^\circ \qquad \ldots(2)$$

Substituting $x^\circ = y^\circ + 18$ in equation (1), we get
$$y^\circ + 18^\circ + y^\circ = 180^\circ$$
$$2y^\circ = 180^\circ - 18^\circ$$
$$y^\circ = \frac{162^\circ}{2^\circ}$$
$$y^\circ = 81^\circ$$

Substituting $y^\circ = 81^\circ$ in equation (2), we get
$$x^\circ = 81^\circ + 18^\circ$$
$$x^\circ = 99^\circ$$

Answer:
The angles are 99° and 81°.

(i) Known:
The cost of 7 bats and 6 balls is ₹ 3800 and the cost of 3 bats and 5 ball is ₹ 1750

Reasoning:
Assuming the cost of 1 bat as ₹ x and cost of 1 ball as ₹ y, two linear equations can be formed for the Known situation.

Solution:
Let the cost of 1 bat = ₹ x
And the cost of 1 ball = ₹ y
Then,
$$7x + 6y = 3800 \qquad \ldots(1)$$
$$3x + 5y = 1750 \qquad \ldots(2)$$

By solving the equation (1)
$$7x + 6y = 3800$$
$$6y = 3800 - 7x$$
$$y = \frac{3800 - 7x}{6} \qquad \ldots(3)$$

Substituting $y=\frac{3800-7x}{6}$ in equation (2), we get

$$3x+5\left(\frac{3800-7x}{6}\right)=1750$$
$$\frac{18x+19000-35x}{6}=1750$$
$$-17x+19000=1750\times 6$$
$$17x=19000-10500$$
$$x=\frac{8500}{17}$$
$$x=500$$

Substituting $x=500$ in equation (3), we get

$$y=\frac{3800-7\times 500}{6}$$
$$y=\frac{300}{6}$$
$$y=50$$

Answer:
Cost of 1 bat is ₹ 500
Cost of 1 ball is ₹ 50

(i) Known:
Taxi Charge for a distance of 10 km is ₹105 and for 15 km is ₹155.

Reasoning:
Assuming fixed charge as ₹ x and charge for each kilometer as ₹ y, two linear equations can be formed.

Solution:
Let the fixed charge = ₹ x
And charge per km = ₹ y

Charge for a distance of 10 km

$$x+10y=105 \qquad \ldots(1)$$

Charge for a distance of 15 km

$$x+15y=155 \qquad \ldots(2)$$

By solving the equation (1)

$$x+10y=105$$
$$x=105-10y \qquad \ldots(3)$$

Substituting $x = 105 - 10y$ in equation (2), we get

$$105 - 10y + 15y = 155$$
$$5y = 155 - 105$$
$$y = \frac{50}{5}$$
$$y = 10$$

Substituting $x = 5$ in equation (3)

$$x = 105 - 10 \times 10$$
$$x = 105 - 100$$
$$x = 5$$

Now, charge for a distance of 25 km $= x + 25y$

$$= 5 + 25 \times 10$$
$$= 5 + 250$$
$$= 255$$

Answer:

Fixed charge = ₹ 5
Charge per km = ₹ 10
Charge for 25 km = ₹ 255

(v) Known:

Fraction becomes $\frac{9}{11}$, if 2 is added to both numerator and denominator and becomes $\frac{5}{6}$, if 3 is added to both numerator and denominator.

Reasoning:

Assuming the numerator as *x* and denominator as *y*, two linear equations can be formed.

Solution:

Let the numerator $= x$
And denominator $= y$

Then fraction $= \frac{x}{y}$

When 2 is added to both numerator and denominator

$$\frac{x+2}{y+2} = \frac{9}{11}$$
$$11(x+2) = 9(y+2)$$
$$11x + 22 = 9y + 18$$
$$11x - 9y + 22 - 18 = 0$$
$$11x - 9y + 4 = 0 \qquad \ldots(1)$$

When 3 is added to both numerator and denominator

$$\frac{x+3}{y+3}=\frac{5}{6}$$
$$6(x+3)=5(y+3)$$
$$6x+18=5y+15$$
$$6x-5y+18-15=0$$
$$6x-5y+3=0 \qquad \ldots(2)$$
$$5y=6x+3$$
$$y=\frac{6x+3}{5} \qquad \ldots(3)$$

Substituting $y=\frac{6x+3}{5}$ in equation (1)

$$11x-9\left(\frac{6x+3}{5}\right)+4=0$$
$$\frac{55x-9(6x+3)+20}{5}=0$$
$$55x-54x-27+20=0$$
$$x-7=0$$
$$x=7$$

Substituting $x=7$ in equation (1)

$$y=\frac{6\times 7+3}{5}$$
$$y=\frac{42+3}{5}$$
$$y=\frac{45}{5}$$
$$y=9$$

Answer:

The fraction is $\frac{7}{9}$

(vi) Known:

Five years hence, the age of Jacob will be three times that of his son and five years ago, Jacob was seven times that of his son.

Reasoning:

Assume their present age as *x* and *y*, then find their age 5 years from now and 5 years ago in terms of *x* and y; two linear equations can be formed.

Solution:

Let the present age of Jacob = *x* years
and his son = *y* years

5 years from now,

Jacob's age $=(x+5)$ years

Son's age $=(y+5)$ years

$$(x+5)=3(y+5)$$
$$x+5=3y+15$$
$$x-3y+5-15=0$$
$$x-3y-10=0 \qquad ...(1)$$

5 years ago,

Jacob's age $=(x-5)$ years

Son's age $=(y-5)$ years

$$(x-5)=7(y-5)$$
$$x-5=7y-35$$
$$x-7y-5+35=0$$
$$x-7y+30=0 \qquad ...(2)$$
$$7\text{y}=x+30$$
$$\text{y}=\frac{x+30}{7} \qquad ...(3)$$

Substituting $\text{y}=\frac{x+30}{7}$ in equation (1)

$$x-3\left(\frac{x+30}{7}\right)-10=0$$
$$\frac{7x-3(x+30)-70}{7}=0$$
$$7x-3x-90-70=0$$
$$4x-160=0$$
$$x=\frac{160}{4}$$
$$x=40$$

Substituting $x=40$ in equation (3)

$$\text{y}=\frac{40+30}{7}$$
$$y=\frac{70}{7}$$
$$y=10$$

Answer:

Present age of Jacob is 40 years and his son is 10 years.

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.4 (Page 56)

Q1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x+y=5$ and $2x-3y=4$

(ii) $3x+4y=10$ and $2x-2y=2$

(iii) $3x-5y-4=0$ and $9x=2y+7$

(iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Difficulty Level: Easy

Unknown:

Solution for the linear pair of equations.

Reasoning

Substitution method:

Pick either of the equations and write one variable in terms of the other then substitute the value of the obtained variable in other equation to solve.

Elimination method:

First multiply one or both the equations by some suitable non-zero constants to make the coefficients of one variable numerically equal then add or subtract one equation from the other so that one variable gets eliminated.

(i) Known:

$$x+y=5$$
$$2x-3y=4$$

Solution

Elimination method:

$$x+y=5 \qquad \ldots(1)$$
$$2x-3y=4 \qquad \ldots(2)$$

Multiplying equation (1) by 2

$$[x+y=5]\times 2$$
$$2x+2y=10 \qquad \ldots(3)$$

By subtracting equation (2) from equation (3)

$$(2x+2y)-(2x-3y)=10-4$$
$$2x+2y-2x+3y=6$$
$$5y=6$$
$$y=\frac{6}{5}$$

Substituting $y=\frac{6}{5}$ in equation (1)

$$x+\frac{6}{5}=5$$

$$x=5-\frac{6}{5}$$

$$x=\frac{25-6}{5}$$

$$x=\frac{19}{5}$$

Substitution method:

$$x+y=5 \qquad \ldots(1)$$

$$2x-3y=4 \qquad \ldots(2)$$

By solving equation (1)

$$x+y=5$$

$$y=5-x \qquad \ldots(3)$$

Substituting $y=5-x$ in equation (2)

$$2x-3(5-x)=4$$

$$2x-15+3x=4$$

$$5x=4+15$$

$$x=\frac{19}{5}$$

Substituting $x=\frac{19}{5}$ in equation (3)

$$y=5-\frac{19}{5}$$

$$y=\frac{25-19}{5}$$

$$y=\frac{6}{5}$$

Answer:

$x=\frac{19}{5}$ $\qquad$ $y=\frac{6}{5}$

(ii) Known:

$$3x+4y=10$$
$$2x-2y=2$$

Solution

Elimination method:

$$3x+4y=10 \qquad \dots(1)$$
$$2x-2y=2 \qquad \dots(2)$$

Multiplying equation (2) by 2

$$[2x-2y=2]\times 2$$
$$4x-4y=4 \qquad \dots(3)$$

By adding equation (1) and equation (3)

$$(3x+4y)+(4x-4y)=10+4$$
$$3x+4y+4x-4y=14$$
$$7x=14$$
$$x=\frac{14}{7}$$
$$x=2$$

Substituting $x=2$ in equation (2)

$$2\times 2-2y=2$$
$$4-2y=2$$
$$2y=4-2$$
$$y=\frac{2}{2}$$
$$y=1$$

Substitution method:

$$3x+4y=10 \qquad \dots(1)$$
$$2x-2y=2 \qquad \dots(2)$$

By solving equation (1)

$$3x+4y=10$$
$$4y=10-3x$$
$$y=\frac{10-3x}{4} \qquad \dots(3)$$

Substituting $y=\frac{10-3x}{4}$ in equation (2)

$$2x-2\left(\frac{10-3x}{4}\right)=2$$

$$\frac{4x-10+3x}{2}=2$$

$$7x-10=4$$

$$7x=4+10$$

$$x=\frac{14}{7}$$

$$x=2$$

Substituting $x=2$ in equation (3)

$$y=\frac{10-3\times 2}{4}$$

$$y=\frac{10-6}{4}$$

$$y=\frac{4}{4}$$

$$y=1$$

Answer:

$x=2 \qquad y=1$

(iii) Known:

$$3x-5y-4=0$$

$$9x=2y+7$$

Solution

Elimination method:

$$3x-5y-4=0 \qquad \ldots(1)$$

$$9x=2y+7 \qquad \ldots(2)$$

Multiplying equation (1) by 3

$$[3x-5y-4=0]\times 3$$

$$9x-15y-12=0 \qquad \ldots(3)$$

By solving equation (2)

$$9x-2y-7=0 \qquad \ldots(4)$$

By subtracting equation (4) from equation (3)

$$(9x-15y-12)-(9x-2y-7)=0$$
$$9x-15y-12-9x+2y+7=0$$
$$-13y-5=0$$
$$-13y=5$$
$$y=-\frac{5}{13}$$

Substituting $y=-\frac{5}{13}$ in equation (2)

$$9x=2\left(-\frac{5}{13}\right)+7$$
$$9x=\frac{-10+91}{13}$$
$$x=\frac{81}{13}\times\frac{1}{9}$$
$$x=\frac{9}{13}$$

Substitution method:

$$3x-5y-4=0 \qquad \ldots(1)$$
$$9x=2y+7 \qquad \ldots(2)$$

By solving equation (1)

$$3x-5y-4=0$$
$$5y=3x-4$$
$$y=\frac{3x-4}{5} \qquad \ldots(3)$$

Substituting $y=\frac{3x-4}{5}$ in equation (2)

$$9x=2\left(\frac{3x-4}{5}\right)+7$$
$$9x=\frac{6x-8+35}{5}$$
$$45x=6x+27$$
$$45x-6x=27$$
$$39x=27$$
$$x=\frac{27}{39}$$
$$x=\frac{9}{13}$$

Substitute $x=\frac{9}{13}$ in equation (3)

$$y=\frac{3\left(\frac{9}{13}\right)-4}{5}$$

$$y=\left(\frac{27-52}{13}\right)\times\frac{1}{5}$$

$$y=-\frac{25}{13}\times\frac{1}{5}$$

$$y=-\frac{5}{13}$$

Answer:

$y=-\frac{5}{13}$ $\quad$ $x=\frac{9}{13}$

(iii) Known:

$$\frac{x}{2}+\frac{2y}{3}=-1$$

$$x-\frac{y}{3}=3$$

Solution

Elimination method:

$$\frac{x}{2}+\frac{2y}{3}=-1 \qquad ...(1)$$

$$x-\frac{y}{3}=3 \qquad ...(2)$$

Multiplying equation (1) by 6 and equation (2) by 3

$$\left[\frac{x}{2}+\frac{2y}{3}=-1\right]\times 6$$

$$3x+4y=-6 \qquad ...(3)$$

$$\left[x-\frac{y}{3}=3\right]\times 3$$

$$3x-y=9 \qquad ...(4)$$

By subtracting equation (4) from equation (3)

$$(3x+4y)-(3x-y)=-6-9$$

$$3x+4y-3x+y=-15$$

$$5y=-15$$

$$y=-\frac{15}{5}$$

$$y=-3$$

Substitute $y=-3$ in equation (2)

$$x-\frac{-3}{3}=3$$
$$x+1=3$$
$$x=3-1$$
$$x=2$$

Substitution method:

$$\frac{x}{2}+\frac{2y}{3}=-1 \qquad \text{...(1)}$$
$$x-\frac{y}{3}=3 \qquad \text{...(2)}$$

By solving equation (2)

$$x-\frac{y}{3}=3$$
$$x=\frac{y}{3}+3$$
$$x=\frac{y+9}{3} \qquad \text{...(3)}$$

Substituting $x=\frac{y+9}{3}$ in equation (1)

$$\frac{1}{2}\left(\frac{y+9}{3}\right)+\frac{2y}{3}=-1$$
$$\frac{y+9+4y}{6}=-1$$
$$5y+9=-6$$
$$5y=-6-9$$
$$y=\frac{-15}{5}$$
$$y=-3$$

Substituting $y=-3$ in equation (3)

$$x=\frac{-3+9}{3}$$
$$x=\frac{6}{3}$$
$$x=2$$

Answer:

$x=2 \qquad y=-3$

Q2. Form the pair of linear equations in the following problems, and find their Solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹. 2000. She asked the cashier to give her ₹. 50 and ₹. 100 notes only. Meena got ₹. 25 notes in all. Find how many notes of ₹. 50 and ₹. 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹. 27 for a book kept for seven days, while Susy paid ₹. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

(i)
Difficulty Level: Medium

Unknown:
The pair of linear equations and fraction

Known:
Fraction becomes 1, if 1 is added to the numerator and 1 is subtracted from the denominator, and fraction becomes $\frac{1}{2}$, if 1 is added to the denominator.

Reasoning:
Fraction has two parts numerator and denominator so assume the numerator as x, and denominator as y, two linear equations can be formed for the known situation.

Solution:

Let the numerator $= x$

And the denominator $= y$

Then the fraction $= \frac{x}{y}$

When 1 is added to the numerator and 1 is subtracted from the denominator;

$$\frac{x+1}{y-1} = 1$$
$$x+1 = y-1$$
$$x+1 = y-1$$
$$x-y+1+1 = 0$$
$$x-y+2 = 0 \qquad \ldots(1)$$

When 1 is added to the denominator;

$$\frac{x}{y+1} = \frac{1}{2}$$
$$2x = y+1$$
$$2x-y-1 = 0 \qquad \ldots(2)$$

By subtracting equation (2) from equation (1)

$$(x-y+2)-(2x-y-1) = 0$$
$$x-y+2-2x+y+1 = 0$$
$$-x+3 = 0$$
$$x = 3$$

Substitute $x = 3$ in equation (1)

$$3-y+2 = 0$$
$$y = 5$$

Answer:

Equations are $x-y+2=0$ and $2x-y-1=0$ where the numerator of the fraction is x, and denominator is y.

Fraction is $\frac{3}{5}$

(ii)

Difficulty Level: Medium

Unknown:

The pair of linear equations and ages of Nuri and Sonu.

Known:

5 years ago, Nuri was thrice as old as Sonu and 10 years later, Nuri will be twice as old as Sonu.

Reasoning:

Assuming the present age of Nuri as x years and Sonu as y years, two linear equations can be formed for the Known Solutions.

Solution:

Let the present age of Nuri $= x$ years

And the present age of Sonu $= y$ years

5 years ago,

Nuri's age $= (x-5)$ years

Sonu's age $= (y-5)$ years

$$x-5=3(y-5)$$
$$x-5=3y-15$$
$$x-3y-5+15=0$$
$$x-3y+10=0 \qquad \ldots(1)$$

10 years later,

Nuri's age $= (x+10)$ years

Sonu's age $= (y+10)$ years

$$x+10=2(y+10)$$
$$x+10=2y+20$$
$$x-2y+10-20=0$$
$$x-2y-10=0 \qquad \ldots(2)$$

By subtracting equation (2) from equation (1)

$$(x-3y+10)-(x-2y-10)=0$$
$$x-3y+10-x+2y+10=0$$
$$-y+20=0$$
$$y=20$$

Figure Substitute $y=20$ in equation (1)

$$x-3\times 20+10=0$$
$$x-60+10=0$$
$$x-50=0$$
$$x=50$$

Answer:

Linear equations are $x-3y+10=0$ and $x-2y-10=0$ where the present age of Nuri is x and Sonu is y.

Age of Nuri is 50 years.

Age of Sonu is 20 years.

(iii)

Difficulty Level: Hard

Unknown:

The pair of linear equations and two-digit number.

Known:

Sum of digits of a two-digit number is 9 and nine times this number twice the number obtained by reversing the order of the digits.

Reasoning:

A two-digits number's form is $10y+x$ where y and x are ten's and one's digit respectively.

Solution:

Let the one's place $=x$

And the ten's place $=y$

Then the number $=10y+x$

Sum of the digits of the number;

$$x+y=9 \qquad \text{...(1)}$$

By reversing the order of the digits, the number $=10x+y$

Hence,

$$9(10y+x)=2(10x+y)$$
$$90y+9x=20x+2y$$
$$20x+2y-90y-9x=0$$
$$11x-88y=0$$
$$11(x-8y)=0$$
$$x-8y=0 \qquad \text{...(2)}$$

By subtracting equation (2) from equation (1)

$$(x+y)-(x-8y)=9-0$$
$$x+y-x+8y=9$$
$$9y=9$$
$$y=1$$

Substitute $y=1$ in equation (1)

$$x+1=9$$
$$x=9-1$$
$$x=8$$

Answer:

Equations are $x + y = 9$ and $8x - y = 0$ where *y* and *x* are ten's and one's digit respectively.

The two-digit number is 18.

(iv)

Difficulty Level: Hard

Unknown:

The pair of linear equations and number of notes of ₹ 50 and ₹ 100 each.

Known:

Meena withdrew ₹ 2000, got ₹ 50 and ₹ 100 notes only and 25 notes in all.

Reasoning:

Assuming the number of notes of ₹ 50 as *x* and ₹ 100 as *y*, two linear equations can be formed for the known Solutions.

Solution:

Let number of notes of ₹ 50 $= x$

and number of notes of ₹ 100 $= y$

Meena got 25 notes in all;

$$x + y = 25 \qquad ...(1)$$

Meena withdrew ₹ 2000;

$$50x + 100y = 2000$$
$$50(x + 2y) = 2000$$
$$x + 2y = \frac{2000}{50}$$
$$x + 2y = 40 \qquad ...(2)$$

By subtracting equation (1) from equation (2)

$$(x + 2y) - (x + y) = 40 - 25$$
$$x + 2y - x - y = 15$$
$$y = 15$$

Substitute $y = 15$ in equation (1)

$$x + 15 = 25$$
$$x = 10$$

Answer:

Equations are $x + y = 25$ and $x + 2y = 40$ where number of ₹ 50 and ₹ 100 notes are *x* and *y* respectively.

Number of ₹ 50 notes is 10

Number of ₹ 100 notes is 15.

(v)

Difficulty Level: Hard

Unknown:

The pair of linear equations, fixed charge and charge for each extra day.

Known:

Saritha paid ₹ 27 for a book kept for 7 days while Susy paid ₹ 21 for a book kept for 5 days, where fixed charge for first 3 days and an additional charge for each day thereafter.

Reasoning:

Assuming fixed charges as ₹ x and additional charge for each extra day as ₹ y, two linear equations can be formed for the known situation.

Solution:

Let the fixed charge $= x$

And charge per extra day $= y$

Saritha paid ₹ 27 for a book kept for 7 days;

$$x+(7-3)y=27$$

$$x+4y=27 \qquad \ldots(1)$$

Susy paid ₹ 21 for a book kept for 5 days;

$$x+(5-3)y=21$$

$$x+2y=21 \qquad \ldots(2)$$

By subtracting equation (2) from equation (1)

$$(x+4y)-(x+2y)=27-21$$

$$x+4y-x-2y=6$$

$$2y=6$$

$$y=\frac{6}{2}$$

$$y=3$$

Substituting $y=3$ in equation (3)

$$x+4\times 3=27$$

$$x+12=27$$

$$x=27-12$$

$$x=15$$

Answer:

Equations are $x+2y=21$ and $x+4y=27$ where fixed charge is ₹ x and charge for each extra day is ₹ y.

Fixed charge is ₹ 15

Charge for each extra day is ₹ 3

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.5

Q1. Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$
$3x-9y-2=0$

(ii) $2x+y = 5$
$3x+2y = 8$

(iii) $3x-5y=20$
$6x-10y=40$

(iv) $x-3y-7=0$
$3x-3y-15=0$

Difficulty Level: Medium

Solution:

(i) $x-3y-3=0$
$3x-9y-2=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\ \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\ \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$
$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other and will not intersect each other thus, there will be no solution for these equations.

Solution:

(ii) $2x+y=5$
$3x+2y=8$

$$2x+y-5=0$$
$$3x+2y-8=0$$
$$\frac{a_1}{a_2}=\frac{2}{3},\ \frac{b_1}{b_2}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-5}{-8}=\frac{5}{8}$$
$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{-8+10}=\frac{y}{-15+16}=\frac{1}{4-3}$$

$$\frac{x}{2}=\frac{y}{1}=1$$

$$\frac{x}{2}=1,\text{ and }\frac{y}{1}=1$$

$$\therefore x=2\text{ and }y=1$$

Solution:

(iii) $3x - 5y = 20$

$6x - 10y = 40$

$$3x-5y-20=0$$

$$6x-10y-40=0$$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{5}{10}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

Solution:

(iv) $x-3y-7=0$

$3x-3y-15=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\ \frac{b_1}{b_2}=\frac{-3}{-3}=1,\ \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$$

$$\frac{a_1}{a_2}\neq\frac{b_l}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$
$$\frac{x}{45-21} = \frac{y}{-21-(-15)} = \frac{1}{-3-(-9)}$$
$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$
$$\frac{x}{24} = \frac{1}{6} \text{ and } \frac{y}{-6} = \frac{1}{6}$$
$$x = 4 \text{ and } y = -1$$
$$\therefore\ x = 4, y = -1$$

Q2.

(i) For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$$2x+3y=7$$
$$(a-b)x+(a+b)y=3a+b-2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x+y=1$$
$$(2k-1)x+(k-1)y=2k+1$$

Difficulty Level: Medium

Solution:

(i) $2x+3y=7$

$(a-b)x+(a+b)y=(3a+b-2)$

$$2x+3y-7=0$$
$$(a-b)x+(a+b)y-(3a+b-2)=0$$
$$\frac{a_1}{a_2}=\frac{2}{a-b},\quad \frac{b_1}{b_2}=\frac{3}{a+b},\quad \frac{c_1}{c_2}=\frac{7}{3a+b-2}$$

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad \ldots(1)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad \ldots(2)$$

Subtracting (1) from (2), we obtain

$$4b=4$$

$$b=1$$

Substituting $b=1$ in equation (2), we obtain

$$a-5\times 1= \ 0$$

$$a=5$$

Hence, $a=5$ and $b=1$ are the values for which the given equations give infinitely many solutions.

Solution:

(ii) $3x+y-1 \ = \ 0$

$(2k-1)x+(k-1)y-2k-1=0$

$$3x+y-1=0$$

$$(2k-1)x+(k-1)y-2k-1=0$$

$$\frac{a_1}{a_2}=\frac{3}{2k-1}, \quad \frac{b_1}{b_2}=\frac{1}{k-1}, \quad \frac{c_1}{c_2}=\frac{-1}{-2k-1}=\frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{2k\text{-}1}=\frac{1}{k\text{-}1}\neq\frac{1}{2k+1}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}$$

$$3k-3=2k-1$$

$$k=2$$

Hence, for $k=2$ the given equation has no solution.

Q3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x+5y=9$$

$$3x+2y=4$$

Difficulty Level: Medium

Solution:

$$8x+5y=9 \qquad \ldots(1)$$

$$3x+2y=4 \qquad \ldots(2)$$

From equation (2), we obtain

$$3x+2y=4$$

$$3x=4-2y$$

$$x=\frac{4-2y}{3} \qquad \ldots(3)$$

Substituting $x=\frac{4-2y}{3}$ in equation (1), we obtain

$$8\left(\frac{4-2y}{3}\right)+5y=9$$

$$\frac{32-16y+15y}{3}=9$$

$$32-y=27$$

$$y=32-27$$

$$y=5$$

Substituting $y=5$ in equation (3), we obtain

$$x=\frac{4-2\times 5}{3}$$

$$x=\frac{-6}{3}$$

$$x=-2$$

$$\text{Hence, } x=-2,\ y=5$$

Again, by cross-multiplication method

$8x+5y=9$
$3x+2y=4$

$$8x+5y-9=0$$
$$3x+2y-4=0$$
$$a_1=8, \quad b_1=5, \quad c_1=-9$$
$$a_2=3, \quad b_2=2, \quad c_2=-4$$
$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$
$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$
$$\frac{x}{-2}=\frac{y}{5}=1$$
$$\frac{x}{-2}=1 \text{ and } \frac{y}{5}=1$$
$$x=-2 \text{ and } y=5$$

Q4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 13 when 1 is subtracted from the numerator and it becomes 14 when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Difficulty Level: Medium

Reasoning:
Assume one variable equal to x and another be y. Then based on given conditions, two linear equations can be formed which can be easily solved.

Solution:
(i)
Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

When a student A, takes food for 20 days, pays ₹ 1000 as hostel charges.

$$x+20y=1000 \qquad \ldots(1)$$

When a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges.

$$x+26y=1180 \qquad \ldots(2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y=180$$
$$y=\frac{180}{6}$$
$$y=30$$

Substituting $y=30$ in equation (1), we obtain

$$x+20\times 30=1000$$

$$x=1000-600$$

$$x=400$$

Answer:
Equations are $x+20y=1000$ and $x+26y=1180$ where x is the fixed charge of the food and y is the charge for food per day

Hence, fixed charge is ₹ 400

And charge per day is ₹ 30

(ii)

Let the numerator be x and denominator be y, thus the fraction be $\frac{x}{y}$

According to the given information,

When 1 is subtracted from the numerator

$$\frac{x-1}{y}=\frac{1}{3}$$

$$3x-3=y$$

$$3x-y=3 \qquad \ldots(1)$$

When 8 is added to the denominator,

$$\frac{x}{y+8}=\frac{1}{4}$$

$$4x=y+8$$

$$4x-y=8 \qquad \ldots(2)$$

Subtracting equation (1) from equation (2), we obtain

$$x=5$$

Putting $x=5$ in equation (1), we obtain

$$3\times 5-y=3$$

$$y=15-3$$

$$y=12$$

Answer:

Equations are $3x-y=3$ and $4x-y=8$ where the numerator of the fraction is *x*, and denominator is *y*.

Hence, the fraction is $\frac{5}{12}$

(iii)

Let the number of right answers and wrong answers be *x* and *y* respectively.

Therefore, total number of questions be $(x+y)$

According to the given information,

$$3x-y=40 \qquad \ldots(1)$$

$$4x-2y=50$$

$$2x-y=25 \qquad \ldots(2)$$

Subtracting equation (2) from equation (1), we obtain

$$x=15 \qquad \ldots(3)$$

Substituting this in equation (2), we obtain

$$2 \times 15 - y = 25$$
$$y = 30 - 25$$
$$y = 5$$

Answer:
Equations are $3x - y = 40$ and $2x - y = 25$ where the number of right and wrong answers are x and y respectively.
number of right answers = 15 and number of wrong answers = 5
Hence, Total number of questions = 20

(iv)
Let the speed of 1st car and 2nd car be u *km/h* and v *km/h* respectively.

According to the given information,

When the cars travel in the same direction at different speeds, they meet in 5 hours.
therefore, distance travelled by 1st car $= 5u\ km$
and distance travelled by 2nd car $= 5v\ km$

$$5u - 5v = 100$$
$$5(u - v) = 100$$
$$u - v = 20 \qquad \ldots(1)$$

When the cars travel towards each other at different speeds, they meet in 1 hour
therefore, distance travelled by 1st car $= u\ km$
and distance travelled by 2nd car $= v\ km$

$$u + v = 100 \qquad \ldots(2)$$

Adding both the equations, we obtain

$$2u = 120$$
$$u = 60$$

Substituting this value in equation (2), we obtain

$$60 + v = 100$$
$$v = 40$$

Answer:
Equations are $u - v = 20$ and $u + v = 100$ where the speed of 1st car and 2nd car be u *km/h* and v *km/h* respectively.
Hence, speed of the 1st car $= 60\ km/h$ and speed of the 2nd car $= 40\ km/h$

(v)
Let length and breadth of rectangle be x unit and y unit respectively.
Then the area of the rectangle be xy square units.

According to the question,

When length is reduced by 5 units and breadth is increased by 3 units, area of the rectangle gets reduced by 9 square units;

$$\begin{aligned}
&(x-5)(y+3)=xy-9\\
&xy+3x-5y-15=xy-9\\
&3x-5y-6=0 \qquad \ldots(1)
\end{aligned}$$

When we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units;

$$\begin{aligned}
&(x+3)(y+2)=xy+67\\
&xy+2x+3y+6=xy+67\\
&2x+3y-61=0 \qquad \ldots(2)
\end{aligned}$$

By cross-multiplication method, we obtain

$$\begin{aligned}
&\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}\\
&\frac{x}{305-(-18)}=\frac{y}{-12-(-183)}=\frac{1}{9-(-19)}\\
&\frac{x}{323}=\frac{y}{171}=\frac{1}{19}\\
&\frac{x}{323}=\frac{1}{19},\quad \frac{y}{171}=\frac{1}{19}\\
&x=17,\quad y=9
\end{aligned}$$

Answer:

Equations are $3x-5y-6=0$ and $2x+3y-61=0$ where length and breadth of the rectangle are x and y respectively.

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.6

Q1. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-1}=2$

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$

$2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Difficulty Level: Medium

Reasoning:

When the variable is in denominator, consider the reciprocal of variable as new variable.

Solution:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows:

$$\frac{1}{2x}+\frac{1}{3y}=2 \Rightarrow \frac{p}{2}+\frac{q}{3}=2 \Rightarrow 3p+2q-12=0 \qquad (1)$$

$$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6} \Rightarrow \frac{p}{3}+\frac{q}{2}=\frac{13}{6} \Rightarrow 2p+3q-13=0 \qquad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5} \text{ and } \frac{q}{15}=\frac{1}{5}$$

$$p=2 \text{ and } q=3$$

Therefore, $\frac{1}{x}=2$ and $\frac{1}{y}=3$

Hence, $x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Substituting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \Rightarrow 2p+3q=2 \qquad (1)$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-14 \Rightarrow 4p-9q=-14 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$$6p+9q=6 \qquad (3)$$

Adding equation (2) and (3), we obtain

$$10p=5$$

$$p=\frac{1}{2}$$

Putting $p=\frac{1}{2}$ in equation (1), we obtain

$$2\times\frac{1}{2}+3q=2$$
$$3q=2-1$$
$$q=\frac{1}{3}$$

Therefore, $p=\frac{1}{\sqrt{x}}=\frac{1}{2}$

$\Rightarrow \sqrt{x}=2$

$\Rightarrow x=4$

And $q=\frac{1}{\sqrt{y}}=\frac{1}{3}$

$\Rightarrow \sqrt{y}=3$

$\Rightarrow y=9$

Hence, $x=4$ and $y=9$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{2x}-4y=23$

Substituting $\frac{1}{x}=p$ in the given equations, we obtain

$$4p+3y=14 \Rightarrow 4p+3y-14=0 \quad (1)$$
$$3p-4y=23 \Rightarrow 3p-4y-23=0 \quad (2)$$

By cross-multiplication, we obtain

$$\frac{p}{-69-56}=\frac{y}{-42-(-92)}=\frac{1}{-16-9}$$
$$\frac{p}{-125}=\frac{y}{50}=\frac{1}{-25}$$
$$\frac{p}{-125}=\frac{1}{-25} \text{ and } \frac{y}{50}=\frac{1}{-25}$$
$$p=5 \text{ and } y=-2$$

Therefore, $p=\frac{1}{x}=5$

$\Rightarrow x=\frac{1}{5}$

Hence, $x=\frac{1}{5}$ and $y=-2$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=2$

Putting $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$ in the given equation, we obtain

$$\frac{5}{x-1}+\frac{1}{y-2}=2 \Rightarrow 5p+q=2 \qquad (1)$$

$$\frac{6}{x-1}-\frac{3}{y-1}=2 \Rightarrow 6p-3q=1 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$$15p+3q=6 \qquad (3)$$

Adding (2) and (3), we obtain

$$21p=7$$

$$p=\frac{1}{3}$$

Putting $p=\frac{1}{3}$ in equation (1), we obtain

$$5\times\frac{1}{3}+q=2$$

$$q=2-\frac{5}{3}$$

$$q=\frac{1}{3}$$

Therefore, $p=\frac{1}{x-1}=\frac{1}{3}$

$\Rightarrow x-1=3$

$\Rightarrow x=4$

and $q=\frac{1}{y-2}=\frac{1}{3}$

$\Rightarrow y-2=3$

$\Rightarrow y=5$

Hence, $x=4$ and $y=5$

(v) $\dfrac{7x-2y}{xy}=5$

$\dfrac{8x+7y}{xy}=15$

$$\frac{7x-2y}{xy} = 5 \Rightarrow \frac{7x}{xy}-\frac{2y}{xy}=5 \Rightarrow \frac{7}{y}-\frac{2}{x}=5 \qquad (1)$$

$$\frac{8x+7y}{xy}=15 \Rightarrow \frac{8x}{xy}+\frac{7y}{xy}=15 \Rightarrow \frac{8}{y}+\frac{7}{x}=15 \qquad (2)$$

Putting $\dfrac{1}{x}=p$ and $\dfrac{1}{y}=q$ in the equations (1) and (2), we obtain

$$\frac{7}{y}-\frac{2}{x}=5 \Rightarrow -2p+7q-5=0 \qquad (3)$$

$$\frac{8}{y}+\frac{7}{x}=15 \Rightarrow 7p+8q-15=0 \qquad (4)$$

By cross-multiplication method, we obtain

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$$\frac{p}{-65}=\frac{1}{-65} \text{ and } \frac{q}{-65}=\frac{1}{-65}$$

$p=1$ and $q=1$

Therefore, $p=\dfrac{1}{x}=1$

$\Rightarrow x=1$

and, $q=\dfrac{1}{y}=1$

$\Rightarrow y=1$

Hence, $x=1$ and $y=1$

(vi) $6x+3y=6xy$

$2x+4y=5xy$

By dividing both the given equations by (xy), we obtain

$$6x+3y=6xy \Rightarrow \frac{6}{y}+\frac{3}{x}=6 \qquad (1)$$

$$2x+4y=5xy \Rightarrow \frac{2}{y}+\frac{4}{x}=5 \qquad (2)$$

Substituting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in the equations (1) and (2), we obtain

$$3p+6q-6=0 \qquad (3)$$

$$4p+2q-5=0 \qquad (4)$$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)}=\frac{q}{-24-(-15)}=\frac{1}{6-24}$$

$$\frac{p}{-18}=\frac{q}{-9}=\frac{1}{-18}$$

$$\frac{p}{-18}=\frac{1}{-18} \text{ and } \frac{q}{-9}=\frac{1}{-18}$$

$$p=1 \text{ and } q=\frac{1}{2}$$

Therefore, $p=\frac{1}{x}=1$

$\Rightarrow x=1$

and, $q=\frac{1}{y}=\frac{1}{2}$

$\Rightarrow y=2$

Hence, $x=1$ and $y=2$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

Substituting $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$ in the given equations, we obtain

$$\frac{10}{x+y}+\frac{2}{x-y}=4 \Rightarrow 10p+2q=4 \Rightarrow 5p+q-2=0 \qquad (1)$$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2 \Rightarrow 15p-5q=-2 \Rightarrow 15p-5q+2=0 \qquad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{2-10}=\frac{q}{-30-10}=\frac{1}{-25-15}$$

$$\frac{p}{-8}=\frac{q}{-40}=\frac{1}{-40}$$

$$\frac{p}{-8}=\frac{1}{-40} \text{ and } \frac{q}{-40}=\frac{1}{-40}$$

$$p=\frac{1}{5} \text{ and } q=1$$

Therefore, $p = \frac{1}{x+y} = \frac{1}{5}$

$\Rightarrow x + y = 5 \qquad (3)$

and, $q = \frac{1}{x-y} = 1$

$\Rightarrow x - y = 1 \qquad (4)$

Adding equation (3) and (4), we obtain

$2x = 6$

$x = 3$

Substituting $x = 3$ in equation (3), we obtain

$3 + y = 5$

$y = 2$

Hence, $x = 3$ and $y = 2$

(*viii*) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Substituting $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$ in these equations, we obtain

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \Rightarrow p + q = \frac{3}{4} \qquad (1)$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8} \Rightarrow \frac{p}{2} - \frac{q}{2} = -\frac{1}{8} \Rightarrow p - q = -\frac{1}{4} \qquad (2)$

Adding (1) and (2), we obtain

$2p = \frac{3}{4} - \frac{1}{4}$

$2p = \frac{1}{2}$

$p = \frac{1}{4}$

Substituting $p = \frac{1}{4}$ in (2), we obtain

$\frac{1}{4} - q = -\frac{1}{4}$

$q = \frac{1}{4} + \frac{1}{4}$

$q = \frac{1}{2}$

Therefore, $p = \frac{1}{3x+y} = \frac{1}{4}$

$\Rightarrow 3x + y = 4 \qquad (3)$

and, $q = \frac{1}{3x-y} = \frac{1}{2}$

$\Rightarrow 3x - y = 2 \qquad (4)$

Adding equations (3) and (4), we obtain

$6x = 6$

$x = 1$

Substituting $x = 1$ in (3), we obtain

$3 \times 1 + y = 4$

$y = 1$

Hence, $x = 1$ and $y = 1$

Q2. Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Difficulty Level: Medium

Reasoning:

Solution:

(i)

Let the Ritu's speed of rowing in still water and the speed of stream be $x\ km/h$ and $y\ km/h$ respectively.

Ritu's speed of rowing;

$$\text{Upstream } = (x-y)\, km/h$$

$$\text{Downstream } = (x+y)\, km/h$$

According to question,

Ritu can row downstream 20 km in 2 hours,

$$2(x+y)=20$$

$$x+y=10 \qquad (1)$$

Ritu can row upstream 4 km in 2 hours,

$$2(x-y)=4$$

$$x-y=2 \qquad (2)$$

Adding equation (1) and (2), we obtain

$$2x=12$$

$$x=6$$

Putting $x=6$ in equation (1), we obtain

$$6+y=10$$

$$y=4$$

Hence, Ritu's speed of rowing in still water is 6 *km/h* and the speed of the current is 4 *km/h*.

(ii)

Let the number of days taken by a woman and a man to finish the work be *x* and *y* respectively.

Therefore, work done by a woman in 1 day $=\frac{1}{x}$

and work done by a man in 1 day $=\frac{1}{y}$

According to the question,

2 women and 5 men can together finish an embroidery work in 4 days;

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4} \qquad (1)$$

3 women and 6 men can finish it in 3 days

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3} \qquad (2)$$

Substituting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in equations (1) and (2), we obtain

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4} \Rightarrow 2p+5q=\frac{1}{4} \Rightarrow 8p+20q-1=0 \quad (3)$$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3} \Rightarrow 3p+6q=\frac{1}{3} \Rightarrow 9p+18q-1=0 \quad (4)$$

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$\frac{p}{-2}=\frac{1}{-36} \text{ and } \frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18} \text{ and } q=\frac{1}{36}$$

Therefore, $p=\frac{1}{x}=\frac{1}{18}$

$\Rightarrow x=18$

and, $q=\frac{1}{y}=\frac{1}{36}$

$\Rightarrow y=36$

Hence, number of days taken by a woman is 18 and by a man is 36.

(iii)
Let the speed of train and bus be u *km/h* and v *km/h* respectively. According to the given information, Roohi travels 300 km and takes 4 hours if she travels 60 km by train and the remaining by bus

$$\frac{60}{u}+\frac{240}{v}=4 \quad (1)$$

If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \quad (2)$$

Substituting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in equations (1) and (2), we obtain

$$\frac{60}{u}+\frac{240}{v}=4 \Rightarrow 60p+240q=4 \quad (3)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \Rightarrow 100p+200q=\frac{25}{6} \Rightarrow 600p+1200q=25 \quad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p+2400q=40 \quad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q = 15$$

$$q = \frac{15}{1200}$$

$$q = \frac{1}{80}$$

Substituting $q = \frac{1}{80}$ in equation (3), we obtain

$$60p + 240 \times \frac{1}{80} = 4$$

$$60p = 4 - 3$$

$$p = \frac{1}{60}$$

Therefore, $p = \frac{1}{u} = \frac{1}{60}$

$\Rightarrow u = 60$

and, $q = \frac{1}{v} = \frac{1}{80}$

$\Rightarrow v = 80$

Hence,
speed of the train $= 60\ km/h$

And speed of the bus $= 80\ km/h$

Chapter 3: Pair of Linear Equations in Two Variables

Exercise 3.7

Q1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Difficulty Level: Medium

Reasoning:

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Solution:

Let the age of Ani and Biju be x and y years respectively.
Therefore, age of Ani's father, Dharam be $2x$ years
And age of Biju's sister Cathy be $\frac{y}{2}$ years

Case (I) When Ani is older than Biju

The ages of Ani and Biju differ by 3 years,

$$x - y = 3 \qquad (1)$$

The ages of Cathy and Dharam differs by 30 years,

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \qquad (2)$$

Subtracting (1) from (2), we obtain

$$3x = 57$$

$$x = 19$$

Substituting $x = 19$ in equation (1), we obtain

$$19 - y = 3$$

$$y = 16$$

Therefore, Ani is 19 years old and Biju is 16 years old

Case (II) When Biju is older than Ani.

The ages of Ani and Biju differ by 3 years,

$$y - x = 3$$

$$-x + y = 3 \qquad (1)$$

The ages of Cathy and Dharam differs by 30 years,

$$2x-\frac{y}{2}=30$$

$$4x-y=60 \qquad (2)$$

Adding (1) and (2), we obtain

$$3x=63$$

$$x=21$$

Substituting $x=21$ in equation (1), we obtain

$$-21+y=3$$

$$y=24$$

Therefore, Ani is 21 years old and Biju is 24 years old.

Hence, Ani is 19 years old and Biju is 16 years old or Ani is 21 years old and Biju is 24 years old.

Q2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II] $\left[\text{Hint: } x+100=2(y-100),\ y+10=6(x-10)\right]$

Difficulty Level: Medium

Reasoning:

Assume the friends have ₹ x and ₹ y with them. Then based on given conditions, two linear equations can be formed which can be easily solved.

Solution:

Let the first friend has ₹ x
And second friend has ₹ y
Using the information given in the question,

When second friend gives ₹ 100 to first friend;

$$x+100=2(y-100)$$

$$x+100=2y-200$$

$$x-2y=-300 \qquad (1)$$

When first friend gives ₹ 10 to second friend;

$$y+10=6(x-10)$$

$$y+10=6x-60$$

$$6x-y=70 \qquad (2)$$

Multiplying equation (2) by 2, we obtain

$$12x - 2y = 140 \qquad (3)$$

Subtracting equation (1) from equation (3), we obtain

$$11x = 440$$
$$x = \frac{440}{11}$$
$$x = 40$$

Substituting $x = 40$ in equation (1), we obtain

$$40 - 2y = -300$$
$$2y = 40 + 300$$
$$y = \frac{340}{2}$$
$$y = 170$$

Therefore, first friend has ₹ 40 and second friend has ₹ 170 with them.

Q3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Difficulty Level: Medium

Known:
Changes in speed of the train as well in the time.

Unknown:
Distance covered by the train.

Reasoning:
Assuming uniform speed of the train be $x\ km/h$ and time taken to travel a given distance be $t\ hours$. Then distance can be calculated by;

$$\text{distance} = \text{speed} \times \text{time}$$

Solution:
Let the uniform speed of the train be $x\ km/h$ and the scheduled time to travel the given distance be $t\ hours$
Then the distance be $xt\ km$

When the train would have been $10\ km/h$ faster, it would have taken $2\ hours$ less than the scheduled time;

$$(x+10)(t-2) = xt$$
$$xt - 2x + 10t - 20 = xt$$
$$-2x + 10t = 20 \qquad (1)$$

When the train were slower by $10\ km/h$, it would have taken $2\ hours$ more than the scheduled time;

$$(x-10)(t+3)=xt$$
$$xt+3x-10t-30=xt$$
$$3x-10t=30 \qquad (2)$$

Adding equations (1) and (2), we obtain

$$x=50$$

Substituting $x=50$ in equation (1), we obtain

$$-2\times 50+10t=20$$
$$-100+10t=20$$
$$10t=120$$
$$t=\frac{120}{10}$$
$$t=12$$

Therefore, distance, $xt=50\times 12=600$

Hence, the distance covered by the train is 600 *km*.

Q4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Difficulty Level: Medium

Known:

Changes in number of students in a row and number of rows.

Unknown:

Number of students in the class.

Reasoning:

Assume number of rows equal to *x* and number of students in each row be *y*. Then the total number of students in the class can be calculated by;

$$\text{total number of students} = \text{number of rows} \times \text{number of students in each row}$$

Solution:

Let the number of rows be x and number of students in each row be y

Then the number of students in the class be xy

Using the information given in the question,

Condition 1 If 3 students are extra in a row, there would be 1 row less

$$(x-1)(y+3)=xy$$
$$xy+3x-y-3=xy$$
$$3x-y=3 \qquad (1)$$

Condition 2 If 3 students are less in a row, there would be 2 rows more

$$(x+2)(y-3)=xy$$
$$xy-3x+2y-6=xy$$
$$-3x+2y=6 \qquad (2)$$

Adding equations (1) and (2), we obtain

$$y=9$$

Substituting $y=9$ in equation (1), we obtain

$$3x-9=3$$
$$3x=12$$
$$x=4$$

Hence, number of students in the class, $xy=4\times9=36$

Q5. In $\Delta ABC, \angle C=3\angle B=2(\angle A+\angle B)$. Find the three angles.

Difficulty Level: Medium

Known:
Relation between the angles of the triangle.

Unknown:
Measurement of each angles of the triangle.

Reasoning:
Sum of the measures of all angles of a triangle is 180°.

Solution:
Let the measurement of $\angle A=x^{o}$
And the measurement of $\angle B=y^{o}$
Using the information given in the question,

$$\angle C=3\angle B=2(\angle A+\angle B)$$
$$\Rightarrow 3\angle B=2(\angle A+\angle B)$$
$$\Rightarrow 3y=2(x+y)$$
$$\Rightarrow 3y=2x+2y$$
$$\Rightarrow 2x-y=0 \qquad (1)$$

We know that the sum of the measures of all angles of a triangle is 180°.
Therefore,

$$\angle A+\angle B+\angle C=180^\circ$$

$$\angle A+\angle B+3\angle B=180^\circ \qquad [\because \angle C=3\angle B]$$

$$\angle A+4\angle B=180^\circ$$

$$x+4y=180 \qquad (2)$$

Multiplying equation (1) by 4, we obtain

$$8x-4y=0 \qquad (3)$$

Adding equations (2) and (3), we obtain

$$9x=180$$

$$x=20$$

Substituting $x=20$ in equation (1), we obtain

$$2\times 20-y=0$$

$$y=40$$

Therefore,

$$\angle A=x^\circ=20^\circ$$

$$\angle B=y^\circ=40^\circ$$

$$\angle C=3\angle B=3\times 40^\circ=120^\circ$$

Q6. Draw graphs of the equations $5x-y=5$ and $3x-y=3$. Determine the coordinates of the vertices of the triangle formed by these lines and the y axis.

Difficulty Level: Medium

Solution:

$$5x-y=5$$

$$\Rightarrow y=5x-5$$

The solution table will be as follows.

x	0	2
y	-5	5

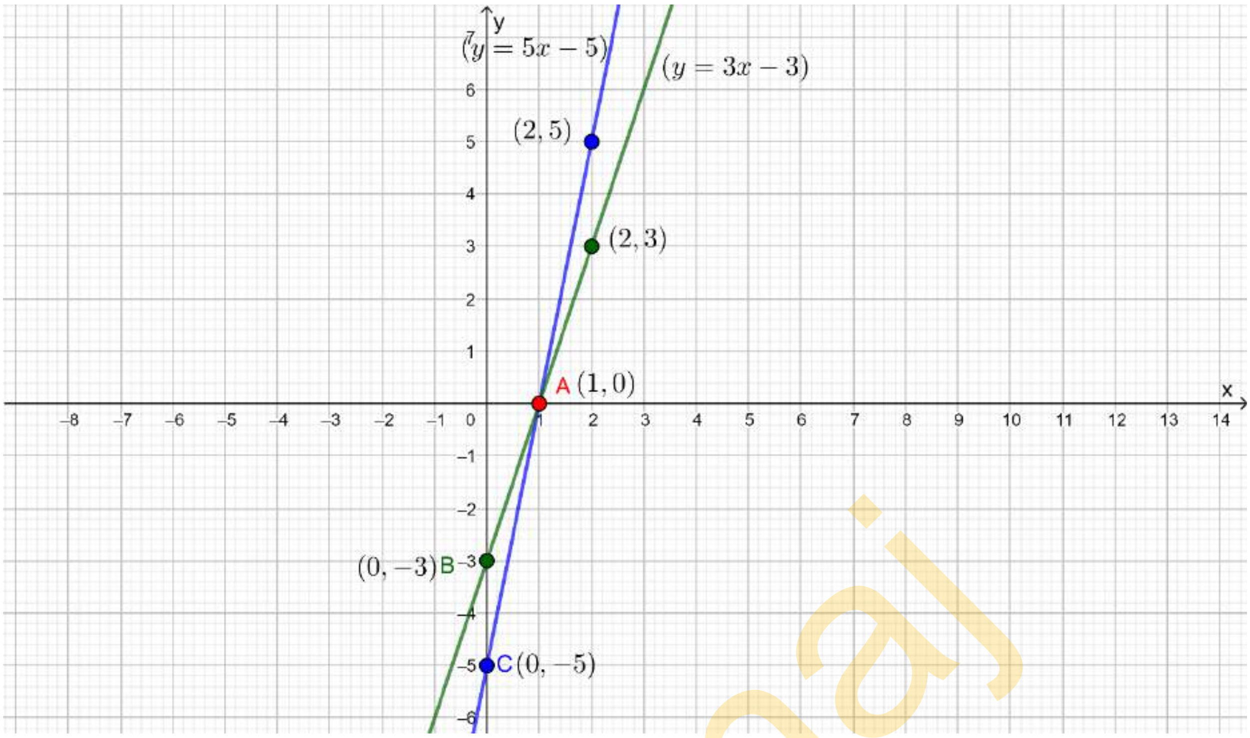
$$3x-y=3$$

$$\Rightarrow y=3x-3$$

The solution table will be as follows.

x	0	2
y	-3	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ABC formed by these lines and y-axis. The coordinates of vertices are A $(1, 0)$, B $(0, -3)$, C $(0, -5)$.

Q7. Solve the following pair of linear equations.

(i) $px + qy = p - q$
$qx - py = p + q$

(ii) $ax + by = c$
$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$
$ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$
$(a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74$
$-378x + 152y = -604$

Difficulty Level: Medium

Solution:

(i) $px + qy = p - q$ ...(1)
$qx - py = p + q$ ...(2)

Multiplying equation (1) by p and equation (2) by q, we obtain

$$p^2x + pqy = p^2 - pq \quad \ldots(3)$$
$$q^2x - pqy = pq + q^2 \quad \ldots(4)$$

Adding equations (3) and (4), we obtain

$$p^2x + q^2x = p^2 + q^2$$

$$\left(p^2 + q^2\right)x = p^2 + q^2$$

$$x = \frac{p^2 + q^2}{p^2 + q^2}$$

$$x = 1$$

Substituting $x = 1$ in equation (1), we obtain

$$p \times 1 + qy = p - q$$

$$qy = -q$$

$$y = -1$$

Therefore, $x = 1$ and $y = -1$

(ii) $ax + by = c$ $\quad\ldots(1)$

$bx + ay = 1 + c$ $\quad\ldots(2)$

Multiplying equation (1) by a and equation (2) by b, we obtain

$$a^2x + aby = ac \quad \ldots(3)$$

$$b^2x + aby = b + bc \quad \ldots(4)$$

Subtracting equation (4) from equation (3),

$$\left(a^2 - b^2\right)x = ac - bc - b$$

$$x = \frac{c(a-b) - b}{a^2 - b^2}$$

Substituting $x = \frac{c(a-b) - b}{a^2 - b^2}$ in equation (1), we obtain

$$ax + by = c$$

$$a\left(\frac{c(a-b) - b}{a^2 - b^2}\right) + by = c$$

$$\frac{ac(a-b) - ab}{a^2 - b^2} + by = c$$

$$by = c - \frac{ac(a-b) - ab}{a^2 - b^2}$$

$$by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2 - b^2}$$

$$by = \frac{abc - b^2c + ab}{a^2 - b^2}$$

$$by = \frac{bc(a-b) + ab}{a^2 - b^2}$$

$$by = \frac{b\left[c(a-b) + a\right]}{a^2 - b^2}$$

$$y = \frac{c(a-b) + a}{a^2 - b^2}$$

Therefore, $x = \dfrac{c(a-b)-b}{a^2-b^2}$ and $y = \dfrac{c(a-b)+a}{a^2-b^2}$

(iii) $\dfrac{x}{a} - \dfrac{y}{b} = 0 \qquad \ldots(1)$

$ax + by = a^2 + b^2 \qquad \ldots(2)$

By solving equation (1), we obtain

$$\frac{x}{a} - \frac{y}{b} = 0$$

$$x = \frac{ay}{b} \qquad \ldots(3)$$

Substituting $x = \dfrac{ay}{b}$ in equation (2), we obtain

$$a \times \left(\frac{ay}{b}\right) + by = a^2 + b^2$$

$$\frac{a^2y + b^2y}{b} = a^2 + b^2$$

$$\left(a^2 + b^2\right)y = b\left(a^2 + b^2\right)$$

$$y = b$$

Substituting $y = b$ in equation (3), we obtain

$$x = \frac{a \times b}{b}$$

$$x = a$$

Therefore, $x = a$ and $y = b$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2 \qquad \ldots(1)$

$(a+b)(x+y) = a^2 + b^2 \qquad \ldots(2)$

By solving equation (2), we obtain

$$(a+b)(x+y) = a^2 + b^2$$

$$(a+b)x + (a+b)y = a^2 + b^2 \qquad \ldots(3)$$

Subtracting equation (3) from (1), we obtain

$$(a-b)x - (a+b)x = \left(a^2 - 2ab - b^2\right) - \left(a^2 + b^2\right)$$

$$\left[(a-b) - (a+b)\right]x = a^2 - 2ab - b^2 - a^2 - b^2$$

$$\left[a - b - a - b\right]x = -2ab - 2b^2$$

$$-2bx = -2b\left(a+b\right)$$

$$x = \left(a+b\right)$$

Substituting $x = (a+b)$ in equation (1), we obtain

$$(a-b)(a+b)+(a+b)y = a^2 - 2ab - b^2$$
$$(a^2-b^2)+(a+b)y = a^2 - 2ab - b^2$$
$$(a+b)y = a^2 - 2ab - b^2 - (a^2-b^2)$$
$$(a+b)y = a^2 - 2ab - b^2 - a^2 + b^2$$
$$y = \frac{-2ab}{(a+b)}$$

(v) $152x - 378y = -74$ $\quad\ldots(1)$

$-378x + 152y = -604$ $\quad\ldots(2)$

Adding equations (1) and (2), we obtain

$$-226x - 226y = -678$$
$$-226(x+y) = -678$$
$$x + y = 3 \quad\ldots(3)$$

Subtracting equation (2) from (1), we obtain

$$530x - 530y = 530$$
$$530(x-y) = 530$$
$$x - y = 1 \quad\ldots(4)$$

Adding equations (3) and (4), we obtain

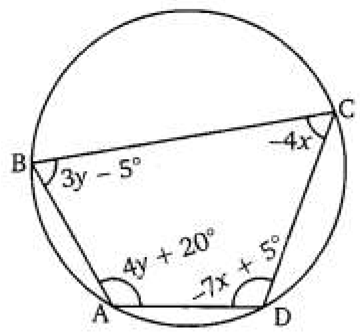
$$2x = 4$$
$$x = 2$$

Substituting $x = 2$ in equation (3), we obtain

$$2 + y = 3$$
$$y = 1$$

Therefore, $x = 2$ and $y = 1$

Q8. ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Difficulty Level: Medium

Known:
Measurement of the angles of the cyclic quadrilateral in terms of *x* and *y*.

Unknown:
Measurement of the angles of the cyclic quadrilateral.

Reasoning:
Pairs of opposite angles of a cyclic quadrilateral are supplementary.

Solution:
We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.
Therefore,

$$\angle A + \angle C = 180^\circ$$
$$(4y+20)+(-4x)=180$$
$$4y+20-4x=180$$
$$-4(x-y)=160$$
$$x-y=-40 \qquad (1)$$

And

$$\angle B + \angle D = 180^\circ$$
$$(3y-5)+(-7x+5)=180$$
$$3y-5-7x+5=180$$
$$-7x+3y=180$$
$$7x-3y=-180 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$$3x-3y=-120 \qquad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$4x=-60$$
$$x=-15$$

Substituting $x=-15$ in equation (1), we obtain

$$-15-y=-40$$
$$y=25$$

Therefore,

$$\angle A = 4\times 25+20=120^\circ$$
$$\angle B = 3\times 25-5=70^\circ$$
$$\angle C = -4\times(-15)=60^\circ$$
$$\angle D = -7\times(-15)+5=110^\circ$$