Chapter- 4: Quadratic Equations

Exercise 4.1 (Page 73 of Grade 10 NCERT)

Q1. Check whether the followings are quadratic equations:

i) $(x+1)^2 = 2(x-3)$

ii) $(x^2 - 2x) = (-2)(3-x)$

iii) $(x-2)(x+1) = (x-1)(x+3)$

iv) $(x-3)(2x+1) = x(x+5)$

v) $(2x-1)(x-3) = (x+5)(x-1)$

vi) $x^2 + 3x + 1 = (x-2)^2$

vii) $(x+2)^3 = 2x(x^2-1)$

viii) $x^3 - 4x - x + 1 = (x-2)^3$

Difficulty Level: Easy

Reasoning:

Standard form of a quadratic equation is $ax^2 + bx + c = 0$ where $a \neq 0$ and degree of quadratic equation is 2.

What is the unknown?

To check whether the given equation is a quadratic equation.

Known:

i) $(x+1)^2 = 2(x-3)$

Solution:

$$\text{Since } (a+b)^2 = a^2 + b^2 + 2ab$$
$$x^2 + 2x + 1 = (2x - 6)$$
$$x^2 + 2x + 1 - (2x - 6) = 0$$
$$x^2 + 2x + 1 - 2x + 6$$
$$x^2 + 7 = 0$$

Here, the degree of $x^2 + 7 = 0$ is 2.

$\therefore$ It is a quadratic equation.

ii) $\left(x^2-2x\right)=(-2)(3-x)$

Solution:

$$x^2-2x=-6+2x$$
$$x^2-2x-2x+6=0$$
$$x^2-4x+6=0$$

Degree = 2.

$\therefore$ It is a quadratic equation.

iii) $(x-2)(x+1)=(x-1)(x+3)$

Solution:

$$x^2-2x+x-2=x^2+3x-x-3$$
$$x^2-x-2=x^2+2x-3$$
$$x^2-x-2-x^2-2x+3=0$$
$$-3x+1=0$$

Degree = 1

$\therefore$ It is not a quadratic equation.

iv) $(x-3)(2x+1)=x(x+5)$

Solution:

$$2x^2+x-6x-3=x^2+5x$$
$$2x^2-5x-3=x^2+5x$$
$$2x^2-5x-3-x^2-5x=0$$
$$x^2-10x-3=0$$

Degree = 2

$\therefore$ It is a quadratic equation.

v) $(2x-1)(x-3)=(x+5)(x-1)$

Solution:

$$2x^2-6x-x+3=x^2-x+5x-5$$
$$2x^2-7x+3=x^2+4x-5$$
$$2x^2-7x+3-x^2-4x+5=0$$
$$x^2-11x+8=0$$

Degree = 2

$\therefore$ It is a quadratic equation

iv) $x^2+3x+1=(x-2)^2$

Solution:

$$x^2+3x+1=x^2-4x+4 \quad \left(\because (a-b)^2=a^2-2ab+b^2\right)$$

$$x^2+3x+1-x^2+4x-4=0$$

$$7x-3=0$$

Degree = 1

$\therefore$ It is not a quadratic equation.

vii) $(x+2)^3=2x(x^2-1)$

Solution:

$$\because (a+b)^3=a^3+3a^2b+3ab^2+b^3$$

$$x^3+3x^2(2)+3(2)^2x+(2)^3=2x^3-2x$$

$$x^3+6x^2+12x+8-2x^3+2x=0$$

$$-x^3+6x^2+14x+8=0$$

Degree = 3

$\therefore$ It is not a quadratic equation.

viii) $x^3-4x^2-x+1=(x-2)^2$

Solution:

$$x^3-4x^2-x+1=x^3-3(x)^2(2)+3(x)(2)^2-(2)^3 \quad \left(\because (a-b)^3=a^3-3a^2b+3ab^2-b^3\right)$$

$$x^3-4x^2-x+1=x^3-6x^2+12x-8$$

$$x^3-4x^2-x+1-x^3+6x^2-12x+8$$

$$2x^2-13x+9=0$$

Degree = 2

$\therefore$ It is a quadratic equation

Q2. Represent the following situation in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

(i) Difficulty Level: Medium

Known:

(i) Area of a rectangular plot is $528 m^2$.

(ii) Length of the plot (in meters) is one more than twice its breadth.

What is the unknown?

Quadratic equation for the situation.

Reasoning:

We know that the area of a rectangle can be expressed as the product of its length and breadth. Since we don't know the length and breadth of the given rectangle, we assume the breadth of the plot to be a variable (x meters). Then, we use the given relationship between the length and breadth: length = 1 + 2 times breadth.

Therefore, Length = $1 + 2x$

Area of rectangle =Length × Breadth

Solution:

Breadth = x

Length = $2x + 1$

Area of Rectangular Plot = Length × Breadth = $528 m^2$

$$(2x + 1) \times (x) = 528$$
$$2x^2 + x = 528$$
$$2x^2 + x - 528 = 0$$

Answers:

Quadratic equation is $2x^2 + x - 528 = 0$, where x is the breadth of the rectangular plot.

(ii) Difficulty Level: Medium

Known:

Product of two consecutive integers is 306.

What is the unknown?
Quadratic equation for the situation.

Reasoning:
Let the first integer be x. Since the integers are consecutive, the next integer is $x+1$.

We know that: First integer × Next integer = 306

Therefore, $x(x+1) = 306$

Solution:

$$x(x+1) = 306$$
$$x^2 + x = 306$$
$$x^2 + x - 306 = 0$$

Quadratic equation is $x^2 + x - 306 = 0$. (Where x is the first integer)

(iii) Difficulty Level: Medium

Known:

(i) Rohan's mother is 26 years older than him.
(ii) Product of their ages 3 year from now will be 360.

What is the unknown?
Quadratic equation for the known situation.

Reasoning:
Let assume that Rohan's present age is x years. Then, from first condition, mother's age is $(x + 26)$ years. Three years from now, Rohan's age will be $x+3$ and Rohan's mother age will be $x + 3 + 26 = x + 29$. The product of their ages is 360.

$$(x+3)\times(x+29) = 360$$

Solution:

$$(x+3)\times(x+29) = 360$$
$$x^2 + 29x + 3x + 87 = 360$$
$$x^2 + 32x + 87 - 360 = 0$$
$$x^2 + 32x - 273 = 0$$

Answers:
Quadratic equation is $x^2 + 32x - 273 = 0$. Where x is the present age of Rohan's age.

(iv) Difficulty Level: Hard

What is the unknown?
Quadratic equation to find the speed of the train.

Known:

(i) Distance covered is 480 km (at a uniform speed).
(ii) If the speed had been 8 km /h less, then it would have taken 3 hours more to cover the same distance.

Reasoning:
Distance is equal to speed multiplied by time. Let the speed be s km/h and time be t hours.

D = st
480 = st

As per the given conditions, for the same distance covered at a speed reduced by 8 km /h, the time taken would have increased by 3 hours. Therefore:

$$(s - 8)(t + 3) = 480 \quad (i)$$

Solution:

$$(s - 8)(t + 3) = 480$$
$$st + 3s - 8t - 24 = 480$$
$$480 + 3s - 8\,(480/s) - 24 = 480$$
$$3s - 3840/s - 24 = 0$$
$$3s(s) - 3840 - 24(s) = 0$$
$$3s^2 - 24s - 3840 = 0$$
$$\frac{3s^2 - 24s - 3840}{3} = 0$$
$$s^2 - 8s - 1280 = 0$$

Answers:
$s^2 - 8s - 1280 = 0$ is the quadratic equation, where s is the speed of the train.

Chapter- 4: Quadratic Equations

Exercise 4.2 (Page 76 of Grade 10 NCERT)

Q1. Find the roots of the following quadratic equations by factorization:

i) $x^2 - 3x - 10 = 0$

ii) $2x^2 + x - 6 = 0$

iii) $\sqrt{(2)}x^2 + 7x + 5\sqrt{2}$

iv) $2x^2 - x + 1/8$

v) $100x^2 - 20x + 1$

Difficulty Level: Medium

Reasoning:

Roots of the polynomial are same as the zeros of the polynomial. Therefore, roots can be found by factorizing the quadratic equation into two linear factors and equating each to zero.

What is the unknown?

Roots of given Quadratic Equations.

What is known?

Quadratic Equations.

i)

Solution:

$$x^2 - 5x + 2x - 10 = 0$$
$$x(x - 5) + 2(x - 5) = 0$$
$$(x - 5)(x + 2) = 0$$
$$x - 5 = 0 \quad \text{and} \quad x + 2 = 0$$
$$x = 5 \qquad x = -2$$

Roots are: $-2, 5$.

ii)

Solution:

$$2x^2 + x - 6 = 0$$
$$2x^2 + 4x - 3x - 6 = 0$$
$$2x(x + 2) - 3(x + 2) = 0$$
$$(2x - 3)(x + 2) = 0$$
$$2x - 3 = 0 \quad \text{and} \quad x + 2 = 0$$
$$2x = 3 \qquad x = -2$$
$$x = 3/2 \quad \text{and} \quad x = -2$$

Roots are: $3/2, -2$

iii)

Solution:

$$\sqrt{(2)}x^2 + 7x + 5\sqrt{2} = 0$$
$$\sqrt{(2)}x^2 + 5x + 2x + 5\sqrt{2} = 0$$
$$\sqrt{(2)}x^2 + 5x + \sqrt{(2)} \times \sqrt{(2)}x + 5\sqrt{2} = 0$$
$$x\left(\sqrt{(2)}x + 5\right) + \sqrt{2}\left(\sqrt{(2)}x + 5\right) = 0$$
$$\left(x\sqrt{(2)} + 5\right)\left(x + \sqrt{2}\right) = 0$$
$$\left(\sqrt{(2)}x + 5\right) = 0 \qquad \left(x + \sqrt{2}\right) = 0$$
$$\sqrt{(2)}x = -5 \qquad x = -\sqrt{2}$$
$$x = (-5)/\sqrt{(2)} \qquad x = -\sqrt{2}$$

Roots are: $(-5)/\sqrt{(2)}, -\sqrt{2}$.

iv)

Solution:

$$2x^2 - x + 1/8 = 0$$

Multiplying both sides of the equation by 8:

$$2(8)x^2 - 8(x) + (8)(1/8) = 0(8)$$
$$16x^2 - 8x + 1 = 0$$
$$16x^2 - 4x - 4x + 1 = 0$$
$$4x(4x - 1) - 1(4x - 1) = 0$$
$$(4x - 1)(4x - 1) = 0$$
$$4x - 1 = 0 \qquad 4x - 1 = 0$$
$$4x = 1 \qquad 4x = 1$$
$$x = 1/4 \qquad x = 1/4$$

Roots are: $1/4, 1/4$.

v)

Solution:

$$100x^2 - 20x + 1 = 0$$
$$100x^2 - 10x - 10x + 1 = 0$$
$$10x(10x - 1) - 1(10x - 1) = 0$$
$$(10x - 1)(10x - 1) = 0$$
$$10x - 1 = 0 \qquad 10x - 1 = 0$$
$$10x = 1 \qquad 10x = 1$$
$$x = 1/10 \qquad x = 1/10$$

Roots are: $1/10, 1/10$.

Q2. Solve the problems given in example 1.

i) John and Jivanti had 45 marbles. Both of them lost 5 marbles each and the product of the no. of marbles they now have is 124. We would like to find out how many marbles they had to start with?

ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

(i) Difficulty Level: Medium

What is the Unknown?

Number of marbles John and Jivanti started with (each).

What is Known?

(i) John and Jivanti together had 45 marbles

(ii) Both of them lost 5 marbles each.

(iii) Product of number of marbles they now have is 124.

Reasoning:

Let the number of marbles that John had be x.

The number of marbles Jivanti had will be (Total marbles MINUS the Number of marbles John had) = 45-x

(i) Both of them lost 5 marbles each:

John = x-5

Jivanti $= 45 - x - 5 = 40 - x$

(ii) Product of current number of marbles=124

$$(x-5)(40-x)=124$$

Solution:

$$\begin{aligned}(x-5)(40-x)&=124\\ 40x-x^2-200+5x&=124\\ -x^2+45x-200-124&=0\\ -x^2+45x-324&=0\\ x^2-45x+324&=0\\ x^2-36x-9x+324&=0\\ x(x-36)-9(x-36)&=0\\ (x-36)(x-9)&=0\end{aligned}$$

$x-36=0 \qquad x-9=0$

$x=36 \qquad x=9$ John and Jivanti started with 36 and 9 marbles.

(ii) Difficulty Level: Hard

What is the Unknown?
Number of toys produced on that day.

What is Known?

(i) Cost of each day is 55 – (number of toys produced in a day) rupees.
(ii) On a particular day the total cost of production was Rs.750.

Reasoning:
Let the number of toys produced in a day is x.

(i) Cost of each toy = (55 - x) rupees
(ii) Total cost of production = cost of each toy × Total number of toys

$$\Rightarrow (55 - x)(x) = 750.$$

Solution:

$$(55 - x)(x) = 750$$
$$55x - x^2 = 750$$
$$55x - x^2 - 750 = 0$$
$$x^2 - 55x + 750 = 0$$
$$x^2 - 25x - 30x + 750 = 0$$
$$x(x - 25) - 30(x - 25) = 0$$
$$(x - 25)(x - 30) = 0$$
$$x - 25 = 0 \qquad x - 30 = 0$$
$$x = 25 \qquad x = 30$$

Number of toys produced on that day is 25 or 30.

Q3. Find two numbers whose sum is 27 and product is 182.

Difficulty Level: Medium

What is the Unknown?
Two numbers.

What is Known?

(i) Sum of two numbers is 27.
(ii) Product of two numbers is 182.

Reasoning:
Let one of the numbers be x. Then the other number will be

x + other number = 27
Other number = 27 – x
Product of the two numbers = 182

This can be written in the form of the following equation:

$$x\,(27 - x) = 182$$

Solution:

$$x\,(27 - x) = 182$$
$$27x - x^2 = 182$$
$$27x - x^2 - 182 = 0$$
$$x^2 - 27x + 182 = 0$$
$$x^2 - 14x - 13x + 182 = 0$$
$$x\,(x - 14) - 13\,(x - 14) = 0$$
$$(x - 13)\,(x - 14) = 0$$
$$x - 13 = 0 \qquad x - 14 = 0$$
$$x = 13 \qquad x = 14$$

The numbers are 13, 14.

Q4. Find two consecutive positive integers, the sum of whose square is 365.

Difficulty Level: Medium

What is the Unknown?
Two consecutive positive integers.

What is Known?
Sum of squares of these two consecutive integers is 365.

Reasoning:
Let the first integer be x.

The next consecutive positive integer will be $x + 1$.

$$x^2 + (x + 1)^2 = 365$$

Solution:

$$x^2 + (x+1)^2 = 365$$
$$x^2 + (x^2 + 2x + 1) = 365 \qquad \because (a+b)^2 = a^2 + 2ab + b^2$$
$$2x^2 + 2x + 1 = 365$$
$$2x^2 + 2x + 1 - 365 = 0$$
$$2x^2 + 2x - 364 = 0$$
$$2(x^2 + x - 182) = 0$$
$$x^2 + x - 182 = 0$$
$$x^2 + 14x - 13x - 182 = 0$$
$$x\,(x + 14) - 13\,(x + 14) = 0$$
$$(x - 13)\,(x + 14) = 0$$
$$x - 13 = 0 \qquad x + 14 = 0$$
$$x = 13 \qquad x = -14$$

Value of x cannot be negative (because it is given that the integers are positive).

$$\therefore x = 13 \qquad x + 1 = 14$$

The two consecutive positive integers are 13 and 14.

Q5. The altitude of right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Difficulty Level: Medium

What is the Unknown?

The measure of the two sides of a given right triangle.

What is Known?

i) Altitude of right triangle is 7 cm less than its base.

ii) Hypotenuse is 13 cm.

Reasoning:

In a right triangle, altitude is one of the sides. Let the base be x cm.

The altitude will be (x - 7) cm.

Next, we can apply the Pythagoras theorem to the given right triangle.

Pythagoras theorem:

$$\text{Hypotenuse}^2 = \text{side } 1^2 + \text{side } 2^2$$
$$13^2 = x^2 + (x - 7)^2$$

Solution:

$$13^2 = x^2 + (x - 7)^2$$
$$169 = x^2 + x^2 - 14x + 49$$
$$169 = 2x^2 - 14x + 49$$
$$2x^2 - 14x + 49 - 169 = 0$$
$$2x^2 - 14x - 120 = 0$$
$$\frac{2x^2 - 14x - 120 = 0}{2}$$
$$x^2 - 7x - 60 = 0$$
$$x^2 - 12x + 5x - 60 = 0$$
$$x(x - 12) + 5(x - 12) = 0$$
$$(x + 5)(x - 12) = 0$$
$$x - 12 = 0 \quad x + 5 = 0$$
$$x = 12 \qquad x = -5$$

We know that the value of the base cannot be negative.

$\therefore$ Base = x = 12 cm

Altitude = 12 -7 = 5 cm

Lengths of two sides are 12 cm and 5 cm.

Q6. A cottage industry produces certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the cost of the production on that day is ₹ 90, find the number of articles produced and the cost of each article.

Difficulty Level: Hard

What is the Unknown?
Number of articles produced and cost of each article.

What is Known?

(i) On a particular day that cost of production of each article (in rupees) was 3 more than twice the no. of articles produced on that day.
(ii) The total cost of the production is ₹ 90.

Reasoning:
Let the number of articles produced on that day be x.
Therefore, the cost (in rupees) of each article will be $(3 + 2x)$

Total cost of production = Cost of each article × Total number of articles

$$90 = (3 + 2x)(x)$$

Solution:

$$90 = (3 + 2x)(x)$$
$$(3 + 2x)(x) = 90$$
$$3x + 2x^2 = 90$$
$$2x^2 + 3x - 90 = 0$$
$$2x^2 + 15x - 12x - 90 = 0$$
$$x(2x + 15) - 6(2x + 15) = 0$$
$$(2x + 15)(x - 6) = 0$$
$$2x + 15 = 0 \qquad x - 6 = 0$$
$$2x = -15 \qquad x = 6$$
$$x = -(15/2) \qquad x = 6$$

Number of articles cannot be a negative number.

$$\therefore x = 6$$

$$\text{Cost of each article} = 3 + 2x = 3 + 2(6) = \text{Rs. } 15$$

Cost of each article is Rs.15.
Number of articles produced is 6.

Chapter- 4: Quadratic Equations

Exercise 4.3 (Page 87 of Grade 10 NCERT)

Q1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square.

i) $2x^2 - 7x + 3 = 0$

ii) $2x^2 + x - 4 = 0$

iii) $4x^2 + 4\sqrt{(3)}x + 3 = 0$

iv) $2x^2 + x + 4 = 0$

Difficulty Level: Medium

What is the unknown?

Roots of the quadratic equation.

Reasoning:

Steps required to solve a quadratic equation by applying the 'completing the square' method are given below:

Let the given quadratic equation be: $ax^2 + bx + c = 0$

(i) Divide all the terms by a

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{bx}{a} + c = 0$$

(ii) Move the constant term $\frac{c}{a}$ to the right side of the equation:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

(iii) Complete the square on the left side of the equation by adding $\frac{b^2}{4a^2}$. Balance this by adding the same value to the right side of the equation.

i)

Solution:

Divide $2x^2 - 7x + 3 = 0$ by 2:

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$x^2 - \frac{7}{2}x = -\frac{3}{2}$$

Since $\frac{7}{2} \div 2 = \frac{7}{4}$, $\left(\frac{7}{4}\right)^2$ should be added to both sides of the equation:

$$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = \frac{-3}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{-3}{2} + \frac{49}{16}$$

$$= \frac{-24+49}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = \left(\pm\frac{5}{4}\right)^2$$

$$x - \frac{7}{4} = \frac{5}{4} \qquad x - \frac{7}{4} = \frac{-5}{4}$$

$$x = \frac{5}{4} + \frac{7}{4} \qquad x = \frac{-5}{4} + \frac{7}{4}$$

$$x = \frac{12}{4} \qquad x = \frac{2}{4}$$

$$x = 3 \qquad x = \frac{1}{2}$$

Roots are $3, \frac{1}{2}$.

ii)

Solution:

$$x^2 + \frac{x}{2} - 2 = 0$$

$$x^2 + \frac{x}{2} = 2$$

$$x^2 + \frac{x}{2} = 2$$

Since, $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\left(\frac{1}{4}\right)^2$ should be added on both sides

$$x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2=2+\left(\frac{1}{4}\right)^2$$

$$\left(x^2+\frac{1}{4}\right)^2=2+\frac{1}{16}$$

$$\left(x^2+\frac{1}{4}\right)^2=\frac{32+1}{16}$$

$$\left(x^2+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\frac{\sqrt{33}}{4}$$

$$\left(x+\frac{1}{4}\right)=\frac{\sqrt{33}}{4} \qquad \left(x+\frac{1}{4}\right)=-\frac{\sqrt{33}}{4}$$

$$x=\frac{\sqrt{33}}{4}-\frac{1}{4} \qquad x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$$

$$x=\frac{\sqrt{33}-1}{4} \qquad x=\frac{-\sqrt{33}-1}{4}$$

Roots are $\frac{\sqrt{33}-1}{4}$, $\frac{-\sqrt{33}-1}{4}$

iii)

$\left(\frac{\sqrt{3}}{2}\right)^2$ *is added on both sides*

$$\left(x+\frac{\sqrt{3}}{2}\right)^2=-\frac{3}{4}+\frac{3}{4}$$

$$\left(x+\frac{\sqrt{3}}{2}\right)^2=0$$

$$x=-\frac{\sqrt{3}}{2} \qquad x=-\frac{\sqrt{3}}{2}$$

Roots are $-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$

iv)

$$x^2+\frac{x}{2}+2=0$$

$$x^2+\frac{x}{2}=-2$$

$$x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2=-2+\left(\frac{1}{4}\right)^2$$

$$\left(x+\frac{1}{4}\right)^2=-2+\frac{1}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{-32+1}{16}$$

$$\left(x+\frac{1}{4}\right)^2=\frac{-31}{16}<0$$

Square of any real number can't be negative.

$\therefore$ Real roots don't exist.

Q2. Find the roots of the quadratic equations given in Q1 above by applying quadratic formula.

Difficulty Level: Medium

What is the Unknown?
Roots of the quadratic equation.

What is Known?
A quadratic equation.

Reasoning:
If the given quadratic equation is : $ax^2 + bx + c = 0$, then:

If $b^2 - 4ac \geq 0$ then the roots are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

If $b^2 - 4ac < 0$ then no real roots exist.

i)
Solution:

$$a=2, b=-7, c=3$$

$$b^2-4ac=(-7)^2-4(2)(3)$$

$$=49-24$$

$$b^2-4ac=25>0$$

$\therefore$ Roots are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(3)}}{2(2)}$$

$$x=\frac{-(-7)\pm\sqrt{49-24}}{2(2)}$$

$$x=\frac{7\pm5}{4}$$

$$x=\frac{7+5}{4}\quad x=\frac{7-5}{4}$$

$$x=\frac{12}{4}\quad x=\frac{2}{4}$$

$$x=3\quad x=\frac{1}{2}$$

Roots are $3,\frac{1}{2}$

ii)

Solution:

$$a=2, b=1, c=-4$$

$$b^2-4ac=(1)^2-4(2)(-4)$$

$$=1+32$$

$$=33>0$$

$$\therefore \text{ Roots are } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-1\pm\sqrt{33}}{2(2)}$$

$$=\frac{-1\pm\sqrt{33}}{4}$$

$$x=\frac{-1+\sqrt{33}}{4}\ x=\frac{-1-\sqrt{33}}{4}$$

Roots are, $\frac{-1+\sqrt{33}}{4},\frac{-1-\sqrt{33}}{4}$

iii)

Solution:

$$a=4, b=(4\sqrt{3}), c=3$$

$$b^2-4ac=(4\sqrt{3})^2-4(4)(3)$$

$$=(16\times3)-(16\times3)$$

$$b^2-4ac=0$$

$$\therefore \quad \text{Roots are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm 0}{2a}$$

$$= \frac{-b}{2a}$$

$$= \frac{-4\sqrt{3}}{2(4)}$$

$$x = \frac{-\sqrt{3}}{2}$$

Roots are $\frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$.

iv)

Solution:

a = 2, b = 1, c = 4

$$b^2 - 4ac = (1)^2 - 4(2)(4)$$
$$= 1 - 32$$
$$= -31 < 0$$
$$b^2 - 4ac < 0$$

$\therefore$ No real roots exist.

Q3. Find the roots of the following equations:

i) $x - \frac{1}{x} = 3, x \neq 0$ ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Difficulty Level: Medium

What is the Unknown?

Roots of a quadratic equation.

What is Known?

Quadratic equation, which is not in the form of $ax^2 + bx + c = 0$

Reasoning:

Convert the given equation in the form of $ax^2 + bx + c = 0$ and by using the quadratic formula, find the roots.

i)

$$x - \frac{1}{x} = 3, x \neq 0$$

Solution:

$x-\frac{1}{x}=3, x\neq 0$ can be rewritten as (multiplying both sides by x):

$$x^2-1=3x$$
$$x^2-3x-1=0$$

Comparing this against the standard form $ax^2+bx+c=0$, we find that:

$$a=1,\ b=-3,\ c=-1$$

$$\begin{aligned}b^2-4ac &= (-3)^2-4(1)(-1)\\ &= 9+4\\ &= 13>0\end{aligned}$$

$$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{13}}{2(1)}$$
$$x=\frac{3\pm\sqrt{13}}{2}$$

The roots are $\frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$.

ii)

Solution:

By cross multiplying we get:

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$$
$$\frac{x-7-x-4}{x^2+4x-7x-28}=\frac{11}{30}$$
$$\frac{-11}{x^2-3x-28}=\frac{11}{30}$$
$$-11\times 30=11\left(x^2-3x-28\right)$$
$$-30=\left(x^2-3x-28\right)$$
$$x^2-3x-28+30=0$$
$$x^2-3x+2=0$$

Comparing this against the standard form $ax^2+bx+c=0$, we find that:

$$a=1,\ b=-3\ c=2$$

$$\begin{aligned}b^2-4ac &= (-3)^2-4(1)(2)\\ &= 9-8\\ &= 1>0\end{aligned}$$

∴ Real roots exist for this quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{3 \pm 1}{2}$$

$$x = \frac{3+1}{2} \qquad x = \frac{3-1}{2}$$

$$x = \frac{4}{2} \qquad x = \frac{2}{2}$$

$$x = 2 \qquad x = 1$$

Roots are 2, 1.

Q4. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Difficulty Level: Medium

What is the Unknown?
Rehman's age.

What is Known?

Sum of reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$.

Reasoning:
Let the present age of Rehman be x years.

3 years ago, Rehman's age was = x - 3

5 years from now age will be = x + 5

Using this information and the given condition, we can form the following equation:

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

Solution:

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

By cross multiplying we get:

$$\frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3}$$
$$\frac{2x+2}{x^2+2x-15}=\frac{1}{3}$$
$$(2x+2)(3)=x^2+2x-15$$
$$6x+6=x^2+2x-15$$
$$x^2+2x-15=6x+6$$
$$x^2+2x-15-6x-6=0$$
$$x^2-4x-21=0$$

Finding roots by factorization:

$$x^2-7x+3x-21=0$$
$$x(x-7)+3(x-7)=0$$
$$(x-7)(x+3)=0$$
$$x-7=0 \qquad x+3=0$$
$$x=7 \qquad x=-3$$

Age can't be a negative value.

$\therefore$ Rehman's present age is 7 year.

Q5. In a class test the sum of Shefali's marks in Mathematics and English is 30. Had She got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Difficulty Level: Medium

What is the Unknown?
Marks of Shefali in two subjects.

What is Known?

(i) Sum of Shefali's marks in Mathematics and English is 30.
(ii) Had she got 2 marks more in Mathematics and 3 marks less in English, product of marks would have been 210.

Reasoning:
Let the marks Shefali scored in mathematics be x.

i) Then, marks scored by her in English $\Rightarrow$ 30 – Marks scored in Mathematics $=30-x$

ii) 2 more marks in Mathematics $=x+2$

iii) 3 marks less in English $= 30 - x - 3$

$= 27 - x$

Product of these two = 210

$$(x + 2)(27 - x) = 210$$

Solution:

$$(x+2)(27-x) = 210$$
$$27x - x^2 + 54 - 2x = 210$$
$$-x^2 + 25x + 54 = 210$$
$$-x^2 + 25x + 54 - 210 = 0$$
$$-x^2 + 25x - 156 = 0$$

Multiplying both sides by -1:

$$x^2 - 25x + 156 = 0$$

Comparing with $ax^2 + bx + c = 0$

$$a = 1, b = -25, c = 156$$
$$b^2 - 4ac = (-25)^2 - 4(1)(156)$$
$$= 625 - 624$$
$$= 1$$
$$b^2 - 4ac > 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(156)}}{2(1)}$$
$$= \frac{-(-25) \pm \sqrt{1}}{2(1)}$$
$$= \frac{25 \pm 1}{2}$$
$$x = \frac{25+1}{2} \quad x = \frac{25-1}{2}$$
$$x = \frac{26}{2} \quad x = \frac{24}{2}$$
$$x = 13 \quad x = 12$$

Two possible answers for this given question:

If Shefali scored 13 mark in mathematics, then mark in English $= (30 - 13) = 17$.

If Shefali scored 12 mark in mathematics, then mark in English $= (30 - 12) = 18$.

Q6. The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the fields.

Difficulty Level: Medium

What is the Unknown?

Sides of rectangular field.

What is Known?

i) The diagonal of the rectangular field is 60 meters more than the shorter side.

ii) The longer side is 30 meters more than the shorter side.

Reasoning:

Let the shorter side be x meter. Then the length of diagonal of field will be x+60 and length of longer side will be x+30. Using Pythagoras theorem, value of x can be found.

By applying Pythagoras theorem:

$$\text{Hypotenuse}^2 = \text{Side } 1^2 + \text{Side } 2^2$$
$$(60+x)^2 = x^2 + (30+x)^2$$

Solution:

$$(60+x)^2 = x^2 + (30+x)^2$$
$$60 + 2(60)x + x^2 = x^2 + 30^2 + 2(30)x + x^2$$
$$3600 + 120x + x^2 = x^2 + 900 + 60x + x^2$$
$$3600 + 120x + x^2 - x^2 - 900 - 60x - x^2 = 0$$
$$2700 + 60x - x^2 = 0$$

$$\because (a+b)^2 = a^2 + 2ab + b^2$$

Multiplying both sides by -1:

$$x^2 - 60x - 2700 = 0$$

Solving by quadratic formula:

Comparing with $ax^2 + bx + c = 0$

$$a = 1,\ b = -60,\ c = -2700$$
$$b^2 - 4ac = (-60)^2 - 4(1)(-2700)$$
$$= 3600 + 10800$$
$$b^2 - 4ac = 14400 > 0$$

$\therefore$ Roots exist.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-60) \pm \sqrt{14400}}{2}$$
$$= \frac{60 \pm \sqrt{14400}}{2}$$
$$x = \frac{60 \pm 120}{2}$$
$$x = \frac{60 + 120}{2} \qquad x = \frac{60 - 120}{2}$$
$$x = \frac{180}{2} \qquad x = \frac{-60}{2}$$
$$x = 90 \qquad x = -30$$

Length can't be a negative value.

$\therefore \; x = 90$

Length of shorter side $x = 90\ m$

Length of longer side $= 30 + x = 30 + 90 = 120\ m$

Q7. The difference of squares of two numbers is 180. The square of smaller number is 8 times the larger number. Find the two numbers.

Difficulty Level: Medium

What is the Unknown?

Two numbers.

What is Known?

i) Difference of squares of two numbers is 180.

ii) The square of the smaller number is 8 times the larger number.

Reasoning:

Let the larger number be x.

Square of smaller number is = $8x$.

Difference of squares of the two numbers is 180.

Square of larger number - Square of smaller number = 180

$$x^2 - 8x - 180 = 0$$

Solution:

$$x^2 - 8x - 180 = 0$$
$$x^2 - 18x + 10x - 180 = 0$$
$$x(x-18) + 10(x-18) = 0$$
$$(x-18)(x+10) = 0$$
$$x - 18 = 0 \qquad x + 10 = 0$$
$$x = 18 \qquad x = -10$$

If the larger number is 18, then square of smaller number $= 8 \times 18$

$$\therefore \text{ Smaller number } = \pm\sqrt{8 \times 18}$$
$$= \pm\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$$
$$= \pm 2 \times 2 \times 3$$
$$= \pm 12$$

If larger number is – 10, then square of smaller number $= 8 \times (-10) = -80$
Square of any number cannot be negative.

$\therefore x = -10$ is not applicable.

The numbers are 18, 12 (or) 18, -12.

Q8. A train travels 360 km at a uniform speed. If the speed had been 5 km /hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Difficulty Level: Hard

What is the Unknown?
Speed of the train.

What is Known?

i) Distance covered by the train at a uniform speed = 360 km

ii) If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey.

Reasoning:
Let the speed of the train be s km/hr and the time taken be t hours.

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$360 = s \times t$$
$$t = \left(\frac{360}{s}\right)$$

Increased speed of the train: $s + 5$

New time to cover the same distance: $t - 1$

$$(s+5)(t-1) = 360 \qquad \ldots(2)$$

Solution:

$$(s+5)(t-1) = 360$$
$$st - s + 5t - 5 = 360$$
$$360 - s + 5\left(\frac{360}{s}\right) - 5 = 360$$
$$-s + \frac{1800}{s} - 5 = 0$$
$$-s^2 + 1800 - 5s = 0$$
$$s^2 + 5s - 1800 = 0$$

Solving by quadratic formula:

Comparing with $ax^2 + bx + c = 0$

$$a = 1,\ b = 5,\ c = -1800$$

$$\begin{aligned} b^2 - 4ac &= (5)^2 - 4(1)(-1800) \\ &= 25 + 7200 \\ &= 7225 > 0 \end{aligned}$$

$\therefore$ Real roots exist.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$s = \frac{-5 \pm \sqrt{7225}}{2}$$
$$s = \frac{-5 \pm 85}{2}$$
$$s = \frac{-5+85}{2}, \quad s = \frac{-5-85}{2}$$
$$s = \frac{80}{2} \quad s = \frac{-90}{2}$$
$$s = 40 \quad s = -45$$

Speed of the train cannot be a negative value.

$\therefore$ Speed of the train is 40 km /hr.

Q9. Two water taps together can fill a tank in $9\ 3/8$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank?

Difficulty Level: Hard

What is the Unknown?

Time taken by smaller tap and larger tap to fill the tank separately.

What is Known?

i) Two water taps together can fill the tank in $9\frac{3}{8}$ hours.

ii) The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately.

Reasoning:

Let the tap of smaller diameter fill the tank in x hours.

Tap of larger diameter takes $(x-10)$ hours.

In x hours, smaller tap fills the tank.

$\therefore$ In one hour, part of tank filled by the smaller tap $= \frac{1}{x}$

In $(x-10)$ hours, larger tap fills the tank.

In one hour, part of tank filled by the larger tap $= \frac{1}{(x-10)}$

In 1 hours, the part of the tank filled by the smaller and larger tap together:

$$\frac{1}{x}+\frac{1}{x-10}$$

$$\therefore \frac{1}{x}+\frac{1}{x-10}=\frac{1}{9\frac{3}{8}}$$

Solution:

$$\frac{1}{x}+\frac{1}{x-10}=\frac{1}{\frac{75}{8}}$$

By taking LCM and cross multiplying:

$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$

$$\frac{2x-10}{x^2-10x}=\frac{8}{75}$$

$$75(2x-10)=8(x^2-10x)$$

$$150x-750x=8x^2-80x$$

$$8x^2-80x-150x+750=0$$

$$8x^2-230x+750=0$$

$$4x^2-115x+375=0$$

Solving by quadratic formula:

Comparing with $ax^2+bx+c=0$

$$a = 4,\ b = -115,\ c = 375$$

$$b^2 - 4ac = (-115)^2-4(4)(375)$$

$$=13225-6000$$

$$=7225$$

$$b^2 - 4ac > 0$$

$\therefore$ Real roots exist.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{115\pm\sqrt{7225}}{8}$$

$$x=\frac{115+85}{8} \qquad x=\frac{115-85}{8}$$

$$x=\frac{200}{8} \qquad x=\frac{30}{8}$$

$$x=25 \qquad x=3.75$$

x cannot be 3.75 hours because the larger tap takes 10 hours less than x

Time taken by smaller tap x = 25 hours

Time taken by larger tap (x - 10) =15 hours.

Q10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of express train is 11km /hr more than that of passenger train, find the average speed of the two trains

Difficulty Level: Hard

What is the Unknown:

Average speed of express train and the passenger train.

What is Known:

i) Express train takes 1 hour less than a passenger train to travel 132 km.

ii) Average speed of express train is 11km/hr more than that of passenger train.

Reasoning:

Let the average speed of passenger train = x km/hr

Average speed of express train = $(x + 11)$ km /hr

Distance = Speed × Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken by passenger train to travel 132 km $= \dfrac{132}{x}$

Time taken by express train to travel 132 km $= \dfrac{132}{x+11}$

Difference between the time taken by the passenger and the express train is 1 hour. Therefore, we can write:

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

Solution:

Solving $\dfrac{132}{x} - \dfrac{132}{x+11} = 1$ by taking the LCM on the LHS:

$$\frac{132(x+11) - 132x}{x(x+11)} = 1$$

$$\frac{132x + 1452 - 132x}{x^2 + 11x} = 1$$

$$1452 = x^2 + 11x$$

$$x^2 + 11x - 1452 = 0$$

By comparing $x^2 + 11x - 1452 = 0$ with the general form of a quadratic equation $ax^2 + bx + c = 0$:

$$a = 1, \quad b = 11, \quad c = -1452$$ a = 1, b = 11, c = - 1452

$$\begin{aligned} b^2 - 4ac &= 11^2 - 4(1)(-1452) \\ &= 121 + 5808 \\ &= 5929 > 0 \end{aligned}$$

$$b^2 - 4ac > 0$$

$\therefore$ Real roots exist.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{5929}}{2(1)} \\ &= \frac{-11 \pm 77}{2} \end{aligned}$$

$$x = \frac{-11 + 77}{2} = \frac{66}{2} = 33 \qquad x = \frac{-11 - 77}{2} = \frac{-88}{2} = -44$$

$$x = 33 \qquad x = -44$$

x can't be a negative value as it represents the speed of the train.

Speed of passenger train = 33 km/hr

Speed of express train =x+11 = 33 + 11 = 44km/hr.

Q11. Sum of the areas of two squares 468 m². If the difference of perimeters is 24m, find the sides of two squares.

Difficulty Level: Hard

What is the Unknown?

Sides of two squares.

What is Known?

i) Sum of the areas of two squares is 468m².

ii) The difference of perimeters is 24m.

Reasoning:

Let the side of first square is x and side of the second square is y.

Area of the square = side × side

Perimeter of the square = 4 × side

Therefore, the area of the first and second square are x^2 and y^2 respectively. Also, the perimeters of the first and second square are $4x$ and $4y$ respectively. Applying the known conditions:

(i) $x^2 + y^2 = 468 \ldots\ldots..(1)$

(ii) $4x - 4y = 24 \ldots\ldots (2)$

Solution:

$$x^2 + y^2 = 468$$
$$4x - 4y = 24$$
$$4(x - y) = 24$$
$$x - y = 6$$
$$x = 6 + y$$

Substitute $x = y + 6$ in equation (1)

$$(y+6)^2 + y^2 = 468$$
$$y^2 + 12y + 36 + y^2 = 468$$
$$2y^2 + 12y + 36 = 468$$
$$2(y^2 + 6y + 18) = 468$$
$$y^2 + 6y + 18 = 234$$
$$y^2 + 6y + 18 - 234 = 0$$
$$y^2 + 6y - 216 = 0$$

Solving by factorization method

$$y^2 + 18y - 12y - 216 = 0$$
$$y(y+18) - 12(y+18) = 0$$
$$(y+18)(y-12) = 0$$
$$y + 18 = 0 \qquad y - 12 = 0$$
$$y = -18 \qquad y = 12$$

y can't be negative value as it represents the side of the square.

Side of the first square $x = y + 6 = 12 + 6 = 18$m

Side of the second square $= 12$m.

Chapter- 4: Quadratic Equations

Exercise 4.4 (Page 91 of Grade 10 NCERT)

Q1. Find the nature of the roots of the following quadratic equations, if the real root exists, find them.

i) $2x^2 - 3x + 5 = 0$

ii) $3x^2 - 4\sqrt{(3)}x + 4 = 0$

iii) $2x^2 - 6x + 3 = 0$

Difficulty Level: Easy

What is the unknown?
Nature of roots.

Reasoning:
The general form of a quadratic equation is $ax^2 + bx + c = 0$

$b^2 - 4ac$ is called the discriminant of the quadratic equation and we can decide whether the real roots are exist or not based on the value of the discriminant:

i) Two distinct real roots, if $b^2 - 4ac > 0$

ii) Two equal real roots, if $b^2 - 4ac = 0$

iii) No real roots, if $b^2 - 4ac < 0$

(i)
Solution:

$$2x^2 - 3x + 5 = 0$$
$$a = 2, \quad b = -3, \quad c = 5 \ \text{a = 2, b = -3 c = 5}$$
$$b^2 - 4ac = (-3)^2 - 4(2)(5)$$
$$= 9 - 40$$
$$= -31$$
$$b^2 - 4ac < 0$$

$\therefore$ No real roots.

ii)

$$3x^2 - 4\sqrt{(3)}x + 4 = 0$$

Solution:

$$a = 3, b = -4\sqrt{(3)}, c = 4$$

$$b^2-4ac=\left(-4\sqrt{(3)}\right)^2-4(3)(4)$$
$$=16\times3-4\times4\times3$$
$$=(16\times3)-(16\times3)$$
$$=0$$
$$b^2-4ac=0$$

$\therefore$ Two equal real roots.

$$\text{Sum of roots } =2x \quad =-\frac{b}{a}$$
$$x=-\frac{b}{2a}$$
$$x=\frac{-(-4\sqrt{3})}{2(3)}$$
$$=\frac{4\sqrt{3}}{2\sqrt{3}\sqrt{3}}$$
$$=\frac{2}{\sqrt{3}}$$

Answer:

Roots are $\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}}$.

iii)

$$2x^2-6x+3=0$$

Solution:

$$a=2,\quad b=-6,\quad c=3$$

$$b^2-4ac=(-6)^2-4(2)(3)$$
$$=36-24$$
$$=12|$$
$$b^2-4ac>0$$

$\therefore$ Two distinct real roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$$
$$=\frac{6\pm\sqrt{12}}{4}$$
$$x=\frac{6\pm2\sqrt{3}}{4}$$
$$x=\frac{3\pm\sqrt{3}}{2}$$

Roots are $x=\frac{3+\sqrt{3}}{2},\frac{3-\sqrt{3}}{2}$.

Q2. Find the value of k for each of the following quadratic equation, so that they have two equal roots.

i) $2x^2 + kx + 3 = 0$

ii) $kx(x - 2) + 6 = 0$

Difficulty Level: Easy

What is the Unknown?

Value of k

What is Known?

Quadratic equation has equal real roots.

Reasoning:

Since the quadratic equation has equal real roots:

Discriminant $b^2 - 4ac = 0$

Solution:

i) $2x^2 + kx + 3 = 0$

$$a = 2, \quad b = k, \quad c = 3$$
$$b^2 - 4ac = 0$$
$$(k)^2 - 4(2)(3) = 0$$
$$k^2 - 24 = 0$$
$$k^2 = 24$$
$$k = \sqrt{24}$$
$$k = \pm\sqrt{2 \times 2 \times 2 \times 3}$$
$$k = \pm 2\sqrt{6}$$

ii) $kx^2 - 2kx + 6 = 0$

$$a = k, \quad b = -2k, \quad c = 6$$
$$b^2 - 4ac = 0$$
$$(-2k)^2 - 4(k)(6) = 0$$
$$4k^2 - 24k = 0$$
$$4k(k - 6) = 0$$
$$k = 6 \quad k = 0$$

If we consider the value of k as 0, then the equation will not longer be quadratic.
Therefore, $k = 6$

Q3. Is it possible to design a rectangular mango groove whose length is twice its breadth, and the area is 800 m^2. If so, find its length and breadth.

Difficulty Level: Medium

What is the Unknown:

Finding the possibility of mango groove and if possible, length and breadth.

What is Known:

i) Mango groove length is twice its breadth.
ii) Area =800m^2.

Reasoning:

Let the breadth of rectangle x m.

Length = $2x$ m

Area = length × breadth

Solution:

Area = length × breadth

$$800 = x \times 2x$$
$$2x^2 = 800$$
$$x^2 = \frac{800}{2}$$
$$x^2 = 400$$
$$x^2 - 400 = 0$$

Discriminant of a quadratic equation is $b^2 - 4ac$. Comparing $x^2 - 400 = 0$ with $ax^2 + bx + c = 0$:

a = 1, b = 0, c = - 400

$$b^2 - 4ac = (0)^2 - 4(1)(-400)$$
$$= +1600 > 0$$

∴ Yes, it is possible to design a mango groove.

$$x^2 - 400 = 0$$
$$x^2 = 400$$
$$x = \pm 20$$

Value of x can't be negative value as it represents the breadth of the rectangle.

∴ $x = 20$ m

So, yes, possible to design the mango groove

Length = $2x = 2(20) = 40$ m

Breadth = $x = 20$ m

Q4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends are 20 years. Four years ago, the product of their age in year was 48 years.

Difficulty Level: Medium

What is the Unknown?
Checking the possibility of the situation and if yes, find the present ages.

Known:

i) The sum of the ages of two friends are 20 years.
ii) Four years ago, the product of their age in year was 48 years.

Reasoning:
Let the age of friend 1 be x years.

Then,

i) Age of friend 2 = 20 – age of friend 1

$= 20 - x$

ii) Four years ago, age of friend 1 $= x - 4$

Four years ago, age of friend 2 $= 20 - x - 4$

Product of their ages: $(x - 4)(20 - x - 4) = 48$

Solution:

$$(x-4)(16-x) = 48$$
$$16x - x^2 - 64 + 4x = 48$$
$$-x^2 + 20x - 64 = 48$$
$$-x^2 + 20x - 64 - 48 = 0$$
$$-x^2 + 20x - 112 = 0$$
$$x^2 - 20x + 112 = 0$$

Let's find the discriminant: $b^2 - 4ac$

$a = 1, \quad b = -20, \quad c = 112$ a = 1, b = -20, c = 112

$$b^2 - 4ac = (-20)^2 - 4(1)(112)$$
$$= 400 - 448$$
$$= -48$$
$$b^2 - 4ac < 0$$

Therefore, there are no real roots. So, this situation is not possible.

Q5. Is it possible to design a rectangular park of perimeter of 80 m and area 400 m^2? If so, find its length and breadth.

Difficulty Level: Medium

What is the Unknown?
Checking the possibility to design a rectangular park with the given condition, If yes find its perimeter.

What is Known?
Perimeter of rectangular park $= 80\ m$

Area of rectangle $= 400\ m^2$

Reasoning:
Perimeter of rectangle $= 2(l+b) = 80 \ldots\ldots(1)$

Area of rectangle $= lb = 400 \qquad \ldots(2)$

We will use the 1st equation to express l in the form of b. Then, we will substitute this value of l in equation 2.

Solution:

$$2(l+b) = 80$$
$$(l+b) = 40$$
$$l = 40 - b$$

Substituting the value of $l = 40 - b$ in equation (2)

$$(40-b)(b) = 400$$
$$40b - b^2 = 400$$
$$40b - b^2 - 400 = 0$$
$$b^2 - 40b + 400 = 0$$

Let's find the discriminant: $b^2 - 4ac$

$$a = 1, b = -40, c = 400$$
$$b^2 - 4ac = (-40)^2 - 4(1)(400)$$
$$= 1600 - 1600$$
$$= 0$$

Therefore, it is possible to design a rectangular park with the given condition:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm 0}{2a}$$

$$= \frac{-(-40)}{2(1)}$$

$$= \frac{40}{2} = 20$$

$$= 20$$

$$b = 20, l = 40 - b = 20$$

Answer:

$\therefore$ Yes, possible to design a rectangular park with side $= 20m$.