Chapter - 6: Triangles

Exercise 6.1 (Page 122 of Grade 10 NCERT)

Q1. Fill in the blanks using the correct word given in brackets:
(i) All circles are ____________. (congruent, similar)
(ii) All squares are ____________. (similar, congruent)
(iii) All ____________ triangles are similar. (isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____________ and (b) their corresponding sides are ____________. (equal, proportional)

(i) Reasoning:
As we know that two similar figures have the same shape but not necessarily the same size. (Same size means radii of the circles are equal)

Solution:
Similar. Since the radii of all the circles are not equal.

(ii) Reasoning:
As we know that two similar figures have the same shape but not necessarily the same size. (same size means sides of the squares are equal.)

Solution:
Similar. Since the sides of the squares are not given equal.

(iii) Reasoning:
An equilateral triangle has equal sides and equal angles .

Solution:
Equilateral. Each angle in an equilateral triangle is 60°.

(iv) Reasoning:
As we know that two polygons of same number of sides are similar if their corresponding angles are equal and all the corresponding sides are in the same ratio or proportion.

i. Since the polygons have same number of sides, we can find each angle using formula, $\left(\frac{2n-4}{n}\times 90^{\circ}\right)$. Here '$n$' is number of sides of a polygon.
ii. We can verify by comparing corresponding sides.

Solution:
(a) Equal
(b) Proportional

Q2. Give two different examples of pair of

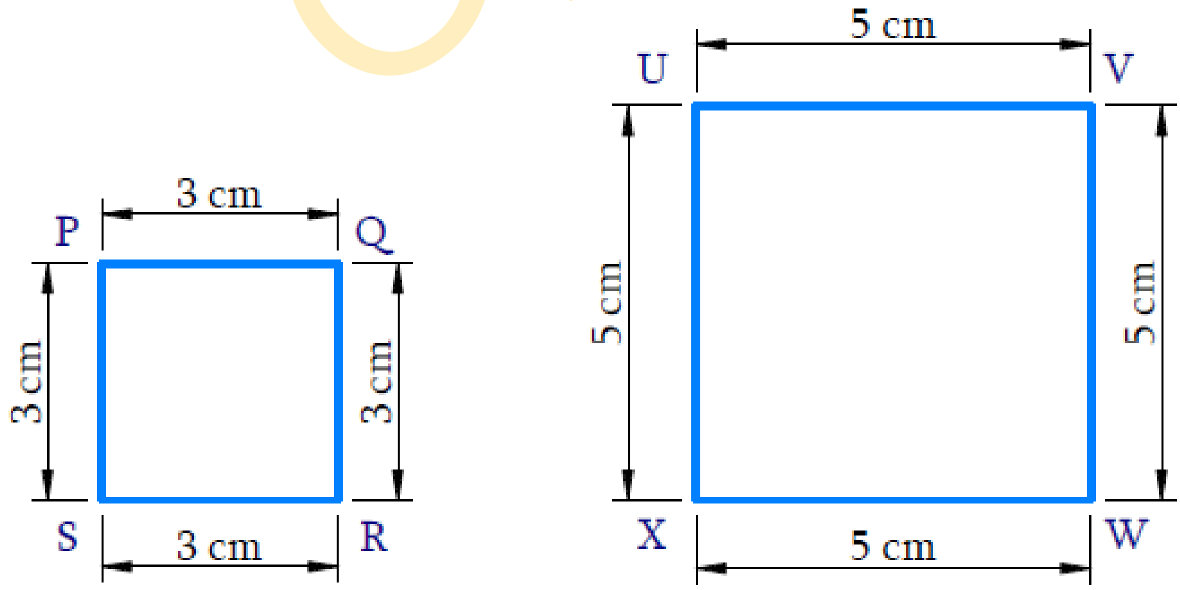
(i) similar figures
(ii) non-similar figures

Solution (i):

(i) Two equilateral triangles of sides 2cm and 6cm.

$\Delta ABC \sim \Delta DEF$ (~ is similar to)

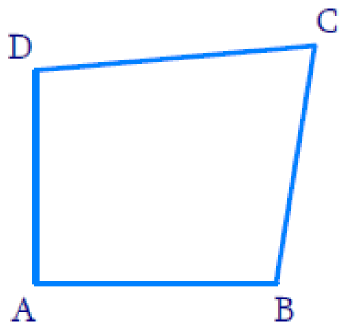
(ii) Two squares of sides 3cm and 5cm.

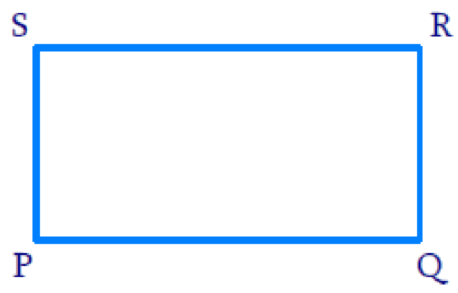


$\square PQRS \sim \square UVWX$ (~ is similar to)

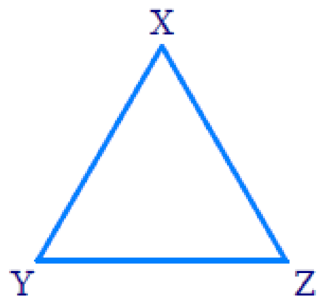
Solution (ii):

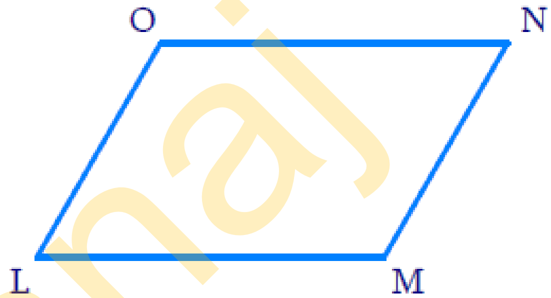
(i) A quadrilateral and a rectangle.



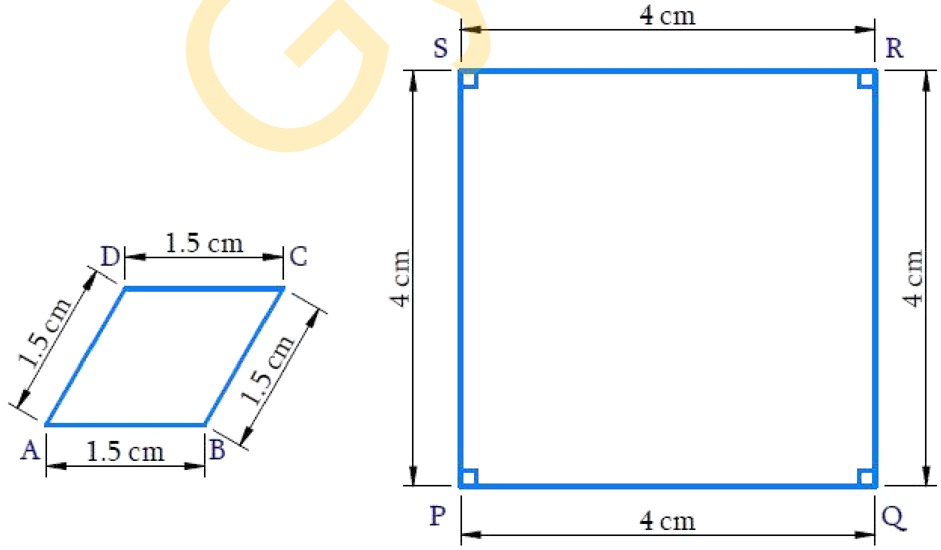


(ii) A triangle and a parallelogram





Q3. State whether the following quadrilaterals are similar or not:



Reasoning:

Two polygons of the same number of sides are similar, if

(i) all the corresponding angles are equal and

(ii) all the corresponding sides are in the same ratio (or proportion).

Solution:

In Quadrilaterals ABCD and PQRS

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{CD}{RS}=\frac{DA}{SP}=\frac{3}{8}$$

$\Rightarrow$ Corresponding sides are in proportion

But $\angle A \neq \angle P; \angle B \neq \angle Q$

$\Rightarrow$ Corresponding angles are not equal

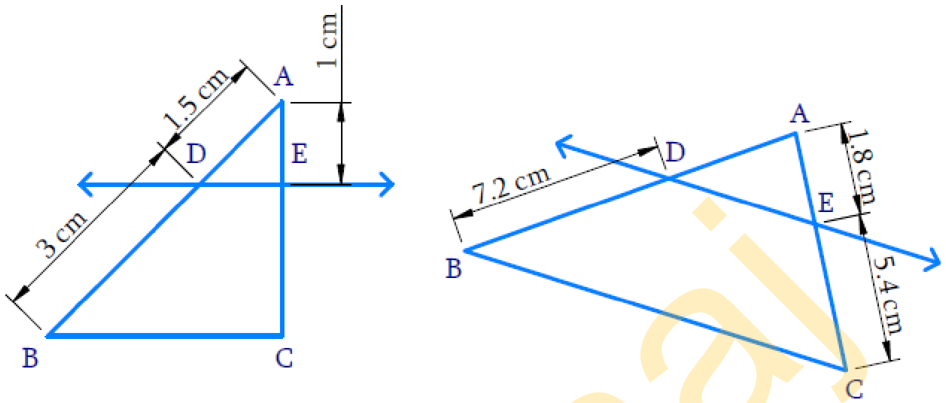
$\square ABCD \nsim \square PQRS$

No, Quadrilateral ABCD is not similar to Quadrilateral PQRS.

Chapter - 6: Triangles

Exercise 6.2 (Page 128 of Grade 10 NCERT)

Q1. In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii)



Reasoning:

As we all know the Basic Proportionality Theorem (B.P.T) or (Thales Theorem)
Two triangles are similar if
(i) Their corresponding angles are equal
(ii) Their corresponding sides are in the same ratio (or proportion)

Solution:

(i) In, ΔABC

$BC \parallel DE$

$$\text{In } \Delta ABC \ \& \ \Delta ADE$$
$$\angle ABC = \angle ADE \ [\because \text{corresponding angles}]$$
$$\angle ACB = \angle AED \ [\because \text{corresponding angles}]$$
$$\angle A = \angle A \text{ common}$$
$$\Rightarrow \Delta ABC \sim \Delta ADE$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{1.5}{3} = \frac{1}{EC}$$
$$EC = \frac{3 \times 1}{1.5}$$
$$EC = 2\text{cm}$$

(ii) Similarly, $\Delta ABC \sim \Delta ADE$

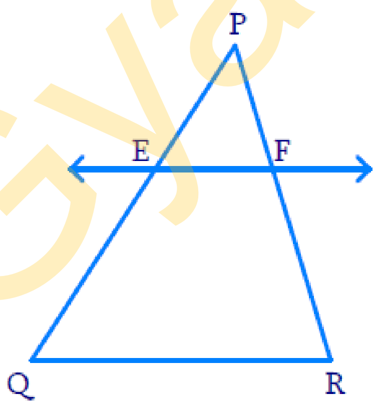
$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$
$$AD = \frac{7.2 \times 1.8}{5.4}$$
$$AD = 2.4\text{cm}$$

Q2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR:

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

(i) Reasoning:
As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

Solution:



Here,
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$
and
$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$
Hence,
$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

According to converse of BPT, EF is not parallel to QR.

(ii) Reasoning:
As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

Solution:
Here,

$$\frac{PE}{EQ}=\frac{4}{4.5}=\frac{8}{9}$$

and

$$\frac{PF}{FR}=\frac{8}{9}$$

Hence,

$$\frac{PE}{EQ}=\frac{PF}{FR}$$

According to converse of BPT, $EF \parallel QR$

(iii) Reasoning:
As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

Solution:
Here,

$$PQ=1.28cm \text{ and } PE=0.18cm$$
$$\begin{aligned} EQ&=PQ-PE\\ &=(1.28-0.18)cm\\ &=1.10cm \end{aligned}$$

$$PR=2.56cm \text{ and } PF=0.36cm$$
$$\begin{aligned} FR&=PR-PF\\ &=(2.56-0.36)cm\\ &=2.20cm \end{aligned}$$

Now,

$$\frac{PE}{EQ}=\frac{0.18cm}{1.10cm}=\frac{18}{110}=\frac{9}{55}$$

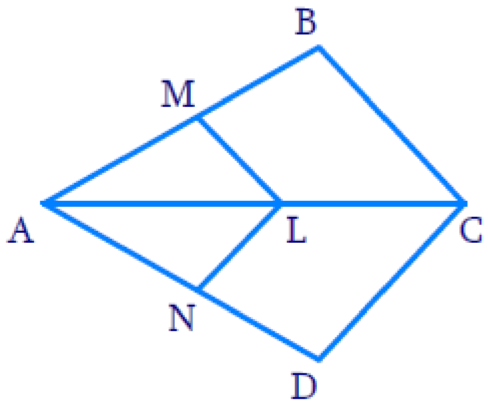
$$\frac{PF}{FR}=\frac{0.36cm}{2.20cm}=\frac{36}{220}=\frac{9}{55}$$

$$\Rightarrow \frac{PE}{EQ}=\frac{PF}{FR}$$

According to converse of BPT, $EF \parallel QR$

Q3. In Fig. 6.18, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Reasoning:

As we know if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In ΔABC

$$LM \parallel CB$$

$$\frac{AM}{MB} = \frac{AL}{LC} \text{...........} \quad (\text{Eq } 1)$$

In ΔACD

$$LN \parallel CD$$

$$\frac{AN}{DN} = \frac{AL}{LC} \text{............} (\text{Eq } 2)$$

From equations (1) and (2)

$$\frac{AM}{MB} = \frac{AN}{DN}$$

$$\Rightarrow \frac{MB}{AM} = \frac{DN}{AN}$$

Adding 1 on both sides

$$\frac{MB}{AM} + 1 = \frac{DN}{AN} + 1$$

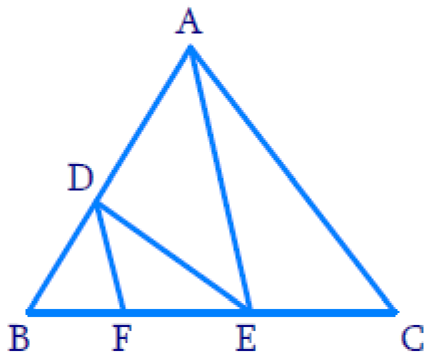
$$\frac{MB + AM}{AM} = \frac{DN + AN}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\frac{AM}{AB} = \frac{AN}{AD}$$

Q4. In Fig. 6.19, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$



Reasoning:

As we know if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In ΔABC

$$DE \parallel AC$$

$$\frac{BD}{AD} = \frac{BE}{EC} \quad ....(i)$$

In ΔABE

$$DF \parallel AE$$

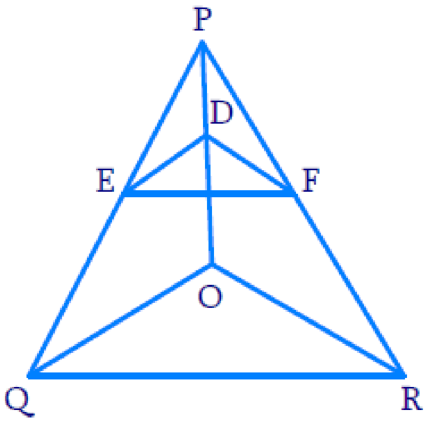
$$\frac{BD}{AD} = \frac{BF}{FE} \quad ....(ii)$$

From (i) and (ii)

$$\frac{BD}{AD} = \frac{BE}{EC} = \frac{BF}{FE}$$

$$\frac{BE}{EC} = \frac{BF}{FE}$$

Q5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.



Reasoning:
As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:
In ΔPOQ

$$DE \parallel OQ \text{(given)}$$
$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \text{..........(1)}$$

In ΔPOR

$$DF \parallel OR\left(given\right)$$
$$\frac{PF}{FR} = \frac{PD}{DO} \text{.........(2)}$$

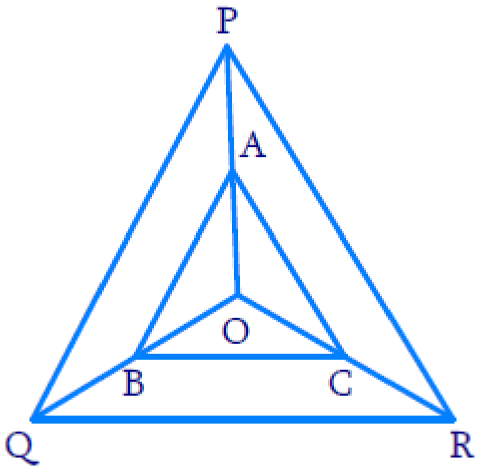
From (1) & (2)

$$\frac{PE}{EQ} = \frac{PF}{FR} = \frac{PD}{DO}$$
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$In\ \Delta PQR$

$$\frac{PE}{EQ} = \frac{PF}{FR}$$
$$\therefore QR \parallel EF \ \left(\text{Converse of BPT}\right)$$

Q6. In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Reasoning:
As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

In ΔOPQ

$$AB \parallel PQ\left(\text{given}\right)$$

$$\frac{OA}{AP}=\frac{OB}{BQ} \quad ....(\text{i})$$

$$\left[\because \text{ Thales Theorem}\left(\text{BPT}\right)\right]$$

In ΔOPR

$$AC \parallel PQ(\text{given})$$

$$\frac{OA}{AP}=\frac{OC}{CR}..............(\text{ii})$$

$$\left[\because \text{ Thales Theorem}\left(\text{BPT}\right)\right]$$

From (i) & (ii)

$$\frac{OA}{AP}=\frac{OB}{BQ}=\frac{OC}{CR}$$

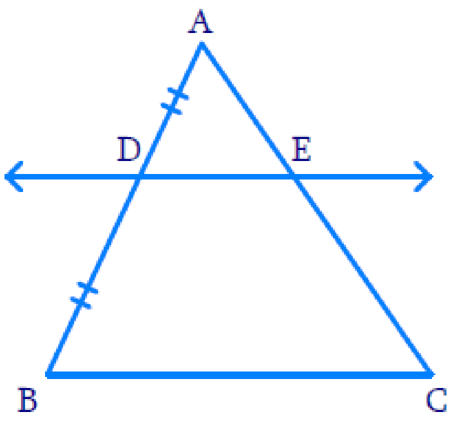
$$\frac{OB}{BQ}=\frac{OC}{CR}$$

Now, In ΔOQR

$$\frac{OB}{BQ}=\frac{OC}{CR}$$

$$BC \parallel QR\left[\because \text{ Converse of BPT}\right]$$

Q7. Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Reasoning:

We know that theorem 6.1 states that "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio (BPT)".

Solution:

In ΔABC , D is the midpoint of AB

Therefore,

$$AD = BD$$

$$\frac{AD}{BD} = 1$$

Now,

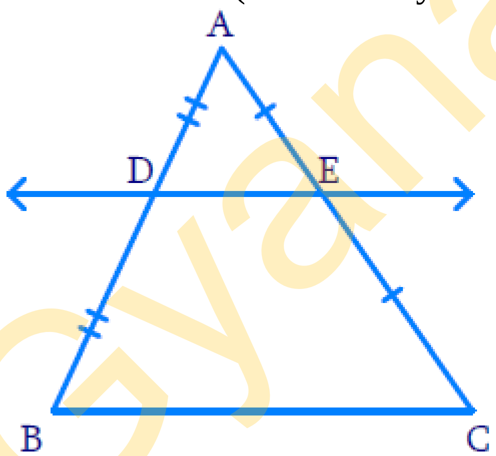
$$DE \parallel BC$$

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{BD} [\text{Theorem } 6.1]$$

$$\Rightarrow \frac{AE}{EC} = 1$$

Hence, E is the midpoint of AC.

Q8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).



Reasoning:

We know that theorem 6.2 tells us if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (Converse of BPT)

Solution:

In ΔABC

D is the midpoint of AB

$$AD = BD$$

$$\frac{AD}{BD} = 1 \text{ ....(i)}$$

E is the midpoint of AC

$$AE = CE$$

$$\frac{AE}{BE} = 1 \text{.........(ii)}$$

From (i) and (ii)

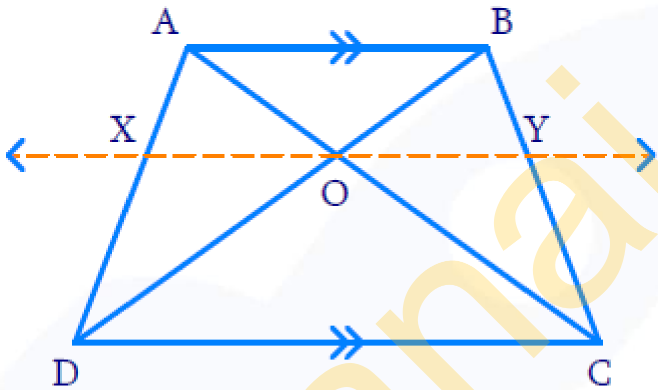
$$\frac{AD}{BD}=\frac{AE}{BE}=1$$

$$\frac{AD}{BD}=\frac{AE}{BE}$$

According to theorem 6.2 (Converse of BPT),

$$DE \parallel BC$$

Q9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO}=\frac{CO}{DO}$



Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

In trapezium ABCD

AB||CD, AC and BD intersect at 'O'

Construct XY parallel to AB and CD (XY||AB, XY||CD) through 'O'

In ΔABC

$$OY \parallel AB \, (\because \text{construction})$$

According to theorem 6.1 (BPT)

$$\frac{BY}{CY}=\frac{AO}{OC}\,\ldots\ldots\ldots\ldots\ldots\ldots(\text{I})$$

In ΔBCD

$$OY \parallel CD \, (\because \text{construction})$$

According to theorem 6.1 (BPT)

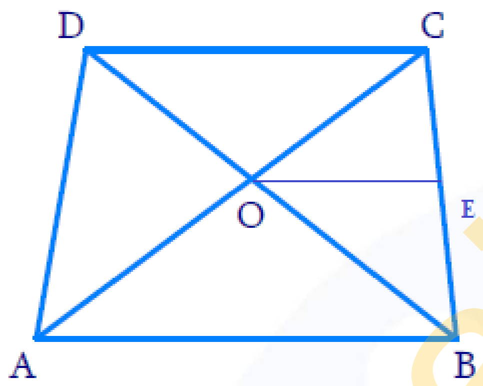
$$\frac{BY}{CY}=\frac{OB}{OD}\,\ldots\ldots\ldots\ldots\ldots\ldots(\text{II})$$

From (I) and (II)

$$\frac{OA}{OC} = \frac{OB}{OD}$$

$$\Rightarrow \frac{OA}{OB} = \frac{OC}{OD}$$

Q10. The diagonals of a quadrilateral ABCD intersect each other at the point 'O' such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.



Reasoning:

As we know If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

In quadrilateral ABCD
Diagonals AC, BD intersect at 'O'

Draw OE||AB

In ΔABC

$$OE \parallel AB$$

$$\Rightarrow \frac{OA}{OC} = \frac{BE}{CE} \ (BPT) ....(1)$$

But $\frac{OA}{OB} = \frac{OC}{OD} \ (given)$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD} ....(2)$$

From (1) and (2)

$$\frac{OB}{OD} = \frac{BE}{CE}$$

In ΔBCD

$$\frac{OB}{OD} = \frac{BE}{CE}$$

$$OE \parallel CD$$

$$OE \parallel AB \text{ and } OE \parallel CD$$

$$\Rightarrow AB \parallel CD$$

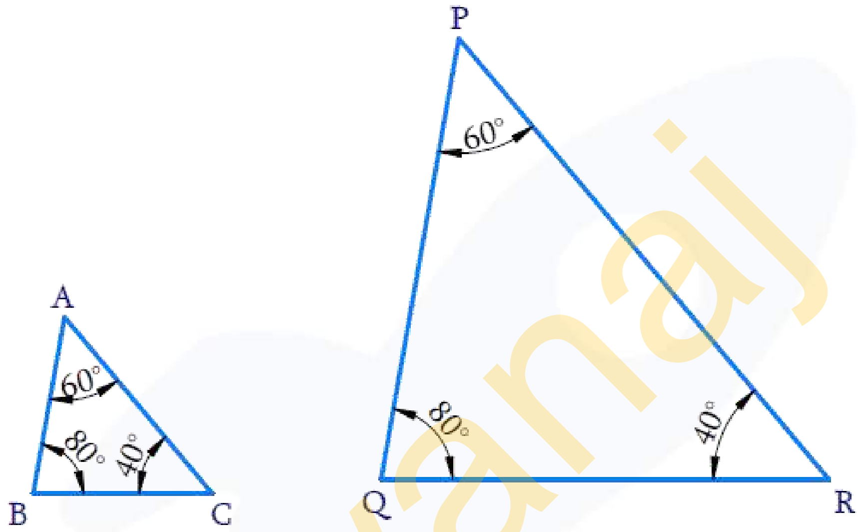
$$\Rightarrow ABCD \text{ is a trapezium}$$

Chapter - 6: Triangles

Exercise 6.3(Page 138)

Q1. State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

1)



Difficulty Level:

Medium

Reasoning:

As we know If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

This is referred as AAA (Angle – Angle – Angle) criterion of similarity of two triangles.

Solution:

In ΔABC and ΔPQR

$\angle A = \angle P = 60^\circ$

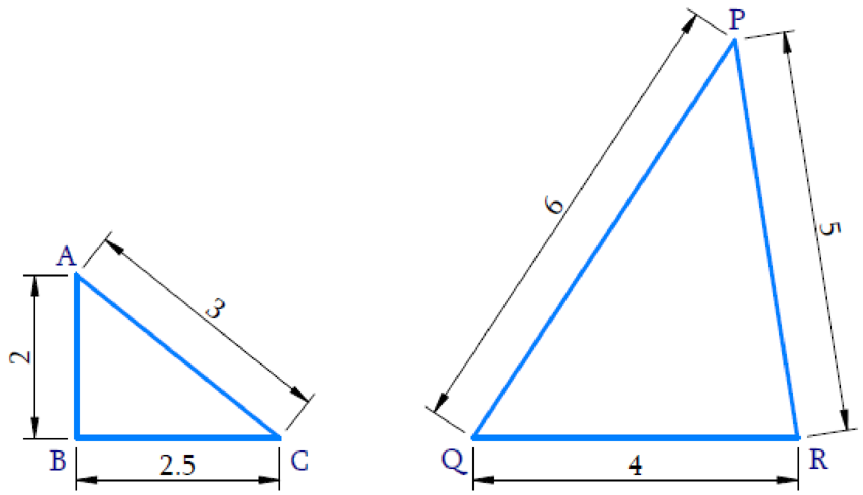
$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

All the corresponding angles of the triangles are equal.

By AAA criterion $\Delta ABC \sim \Delta PQR$

2)



Reasoning:

As we know if in two triangles side of one triangle are proportional to (i.e., in the same ratio of) the side of other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This is referred as SSS (Side – Side – Side) similarity criterion for two triangles.

Solution:

In ΔABC and ΔQRP

$$\frac{AB}{QR}=\frac{2}{4}=\frac{1}{2}$$

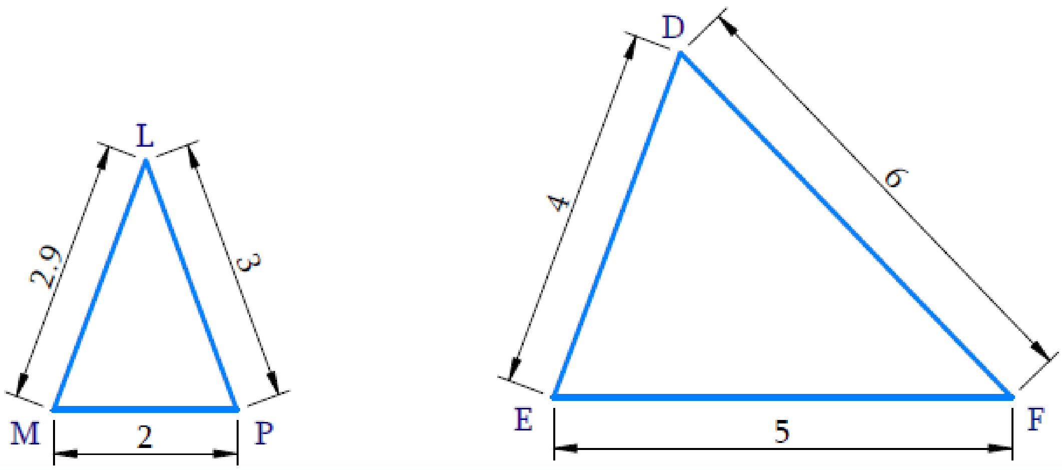
$$\frac{BC}{PR}=\frac{2.5}{5}=\frac{1}{2}$$

$$\frac{AC}{PQ}=\frac{3}{6}=\frac{1}{2}$$

$$\Rightarrow \frac{AB}{QR}=\frac{BC}{PR}=\frac{AC}{PQ}=\frac{1}{2}$$

All the corresponding sides of two triangles are in same proportion.
By SSS criterion $\Delta ABC \sim \Delta QPR$

3)



Reasoning:

As we know if in two triangles side of one triangle are proportional to (i.e., in the same ratio of) the side of other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This is referred as SSS (Side – Side – Side) similarity criterion for two triangles.

Solution:

In ΔLMP and ΔFED

$$\frac{LM}{FE} = \frac{2.7}{5}$$

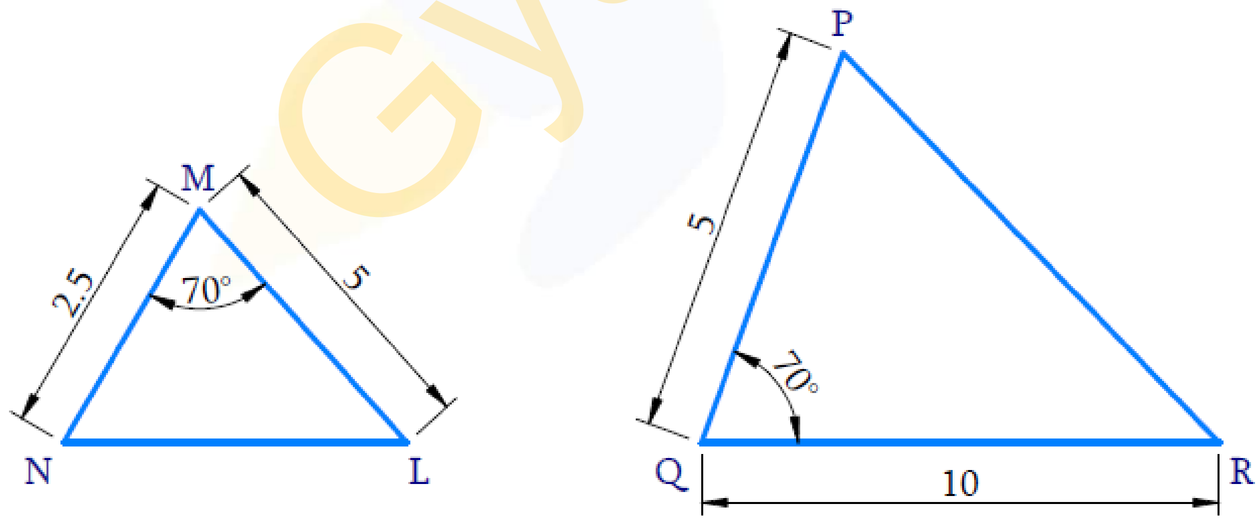
$$\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \frac{LM}{FE} \neq \frac{MP}{ED} = \frac{LP}{FD}$$

All the corresponding sides of two triangles are not in same proportion.
Hence triangles are not similar. $\Delta LMP \nsim \Delta FED$

4)



Reasoning:

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

Solution:

In ΔNML and ΔPQR

$$\frac{NM}{PQ}=\frac{2.5}{5}=\frac{1}{2}$$

$$\frac{ML}{QR}=\frac{5}{10}=\frac{1}{2}$$

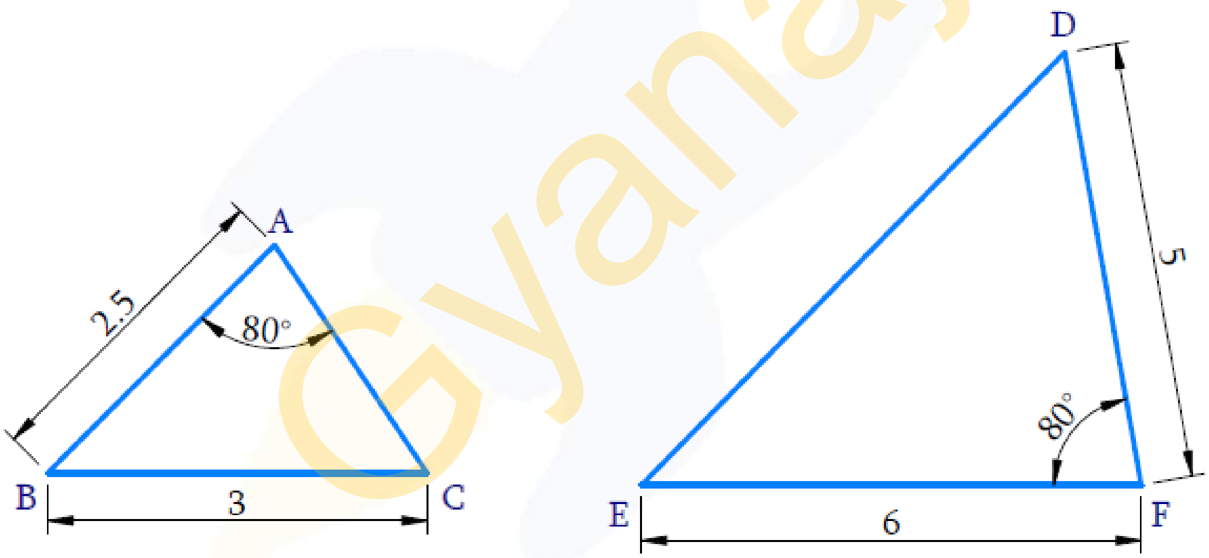
$$\Rightarrow \frac{NM}{PQ}=\frac{ML}{QR}=\frac{1}{2}$$

$$\angle M=\angle Q=70^0$$

One angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional.

By SAS criterion $\Rightarrow$ $\Delta NML \sim \Delta PQR$

5)



Reasoning:

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

Solution:

In ΔABC and ΔDFE

$$\frac{AB}{DF}=\frac{2.5}{5}=\frac{1}{2}$$

$$\frac{BC}{EF}=\frac{3}{6}=\frac{1}{2}$$

$$\frac{AB}{DF}=\frac{BC}{EF}=\frac{1}{2}$$

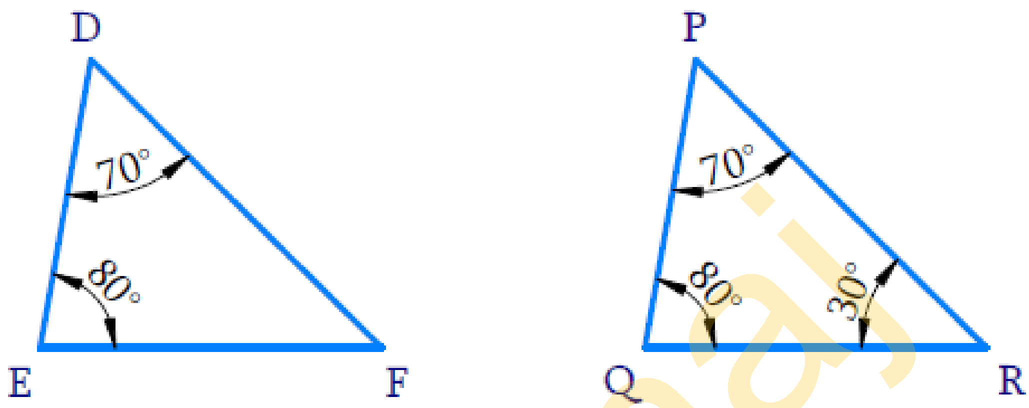
$$\angle A = \angle F = 80^\circ$$

But $\angle B$ must be equal to 80^0

($\because$ The sides AB, BC includes $\angle B$, not $\angle A$)

Therefore, SAS criterion is not satisfied
Hence, the triangles are not similar, $\Delta ABC \nsim \Delta DFE$

6)



Reasoning:
As we know If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

This is referred as AAA (Angle – Angle – Angle) criterion of similarity of two triangles.

Solution:
In ΔDEF

$$\angle D = 70^\circ, \angle E = 80^\circ$$
$$\Rightarrow \angle F = 30^\circ \left[\because \text{Sum of the angles in a triangle is } 180^\circ\right]$$

Similarly,
In ΔPQR

$$\angle Q = 80^\circ, \angle R = 30^\circ$$
$$\Rightarrow \angle P = 70^\circ$$

In ΔDEF and ΔPQR

$$\angle D = \angle P = 70^0$$
$$\angle E = \angle Q = 80^0$$
$$\angle F = \angle R = 30^0$$

All the corresponding angles of the triangles are equal.
By AAA criterion $\Delta DEF \sim \Delta PQR$

Alternate method:

Reasoning:

As we are aware if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

Solution:

In ΔDEF

$$\angle D = 70^{\circ}, \angle E = 80^{\circ}$$

$$\Rightarrow \angle F = 30^{\circ} \left[\because \text{Sum of the angles in a triangle is } 180^{\circ}\right]$$

Now,

In ΔDEF and ΔPQR

$$\angle E = \angle Q = 80^{0}$$

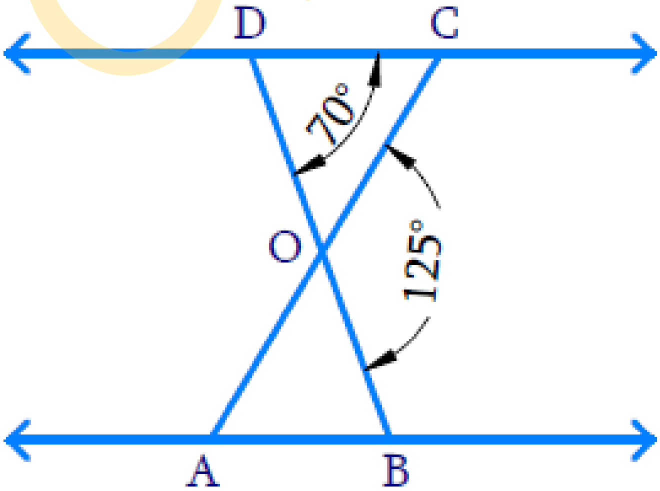
$$\angle F = \angle R = 30^{0}$$

Pair of corresponding angles of the triangles are equal.

By AA criterion $\Delta DEF \sim \Delta PQR$

Q2. In Figure 6.35 $\Delta ODC \sim \Delta OBA$, ∠ BOC = 125°and ∠ CDO = 70°. Find ∠DOC, ∠DCO and ∠OAB.

Diagram



Reasoning:

As we are aware if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

Solution:

In the given figure.

$$\angle DOC = 180^\circ - \angle COB \quad [\because \angle DOC \text{ and } \angle COB \text{ from a linear pair}]$$

$$\angle DOC = 180^\circ - 125^\circ$$

$$\angle DOC = 55^\circ$$

In ΔODC

$$\angle DCO = 180^\circ - (\angle DOC + \angle ODC) \quad [\because \text{angle sum property}]$$

$$\angle DCO = 180 - (55 + 70)$$

$$\angle DCO = 55^\circ$$

In ΔODC and ΔOBA

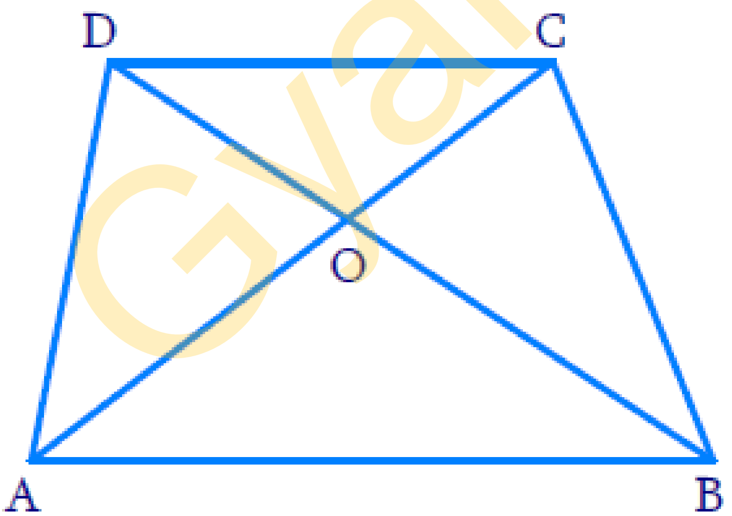
$$\Delta ODC \sim \Delta OBA$$

$$\Rightarrow \angle DCO = \angle OAB$$

$$\angle OAB = 55^\circ$$

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Diagram



Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

Solution

In ΔAOB and ΔCOD

$$\angle AOB = \angle COD \quad (\text{vertically opposite angles})$$

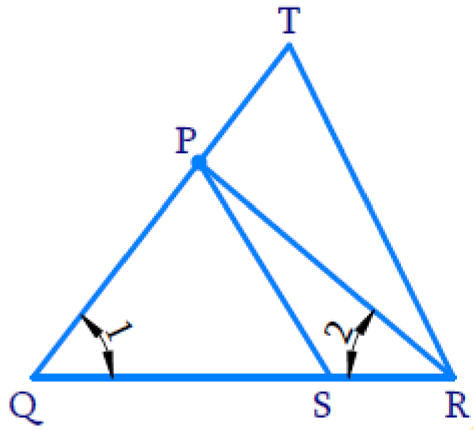
$$\angle BAO = \angle DCO \quad (\text{alternate interior angles})$$

$$\Rightarrow \Delta AOB \sim \Delta COD \quad (\text{AA criterion})$$

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

Q4. In Figure 6.36 $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.

Diagram



Reasoning:

As we know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

Solution

In ΔPQR

$$\angle 1 = \angle 2$$

$\Rightarrow PR = PQ$ (In a triangle sides opposite to equal angles are equal)

In ΔPQS and ΔTQR

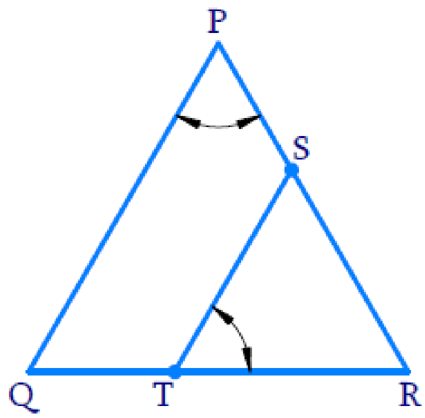
$\angle PQS = \angle TQR = \angle 1$ (same angle)

$\frac{QR}{QS} = \frac{QT}{PQ}$ ($\because PR = PQ$)

$\Rightarrow \Delta PQS \sim \Delta TQR$ ($\because$ SAS criterion)

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Diagram



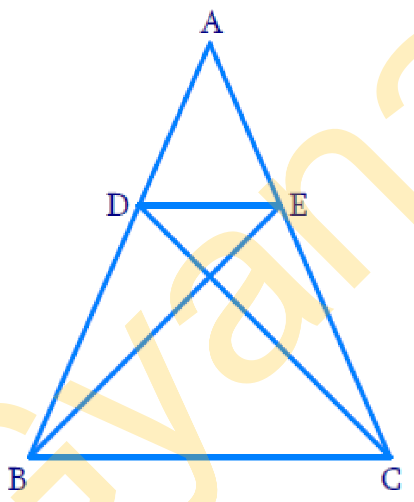
Reasoning:
If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution
In $\Delta RPQ, \Delta RTS$

$$\angle RPQ = \angle RTS \quad \text{(given)}$$
$$\angle PRQ = \angle TRS \quad \text{(common angle)}$$
$$\Rightarrow \Delta RPQ \sim \Delta RTS \quad (\because \text{AA criterion})$$

Q6. In Figure 6.37, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.

Diagram



Reasoning:
As we know if two triangles are congruent to each other; their corresponding parts are equal.
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This is referred as SAS (Side – Angle – Side) similarity criterion for two triangles.

Solution
In ΔABE and ΔACD

$$AE = AD \; (\because \Delta ABE \cong \Delta ACD \; given) ....(1)$$
$$AB = AC \; (\because \Delta ABE \cong \Delta ACD \; given) ....(2)$$

Now Consider $\Delta ADE, \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{from } (1) \,\&\, (2)$$
$$\text{and } \angle DAE = \angle BAC \quad \text{(Common angle)}$$
$$\Rightarrow \Delta ADE \sim \Delta ABC \; \text{(SAS criterion)}$$

Q7. In Figure 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:

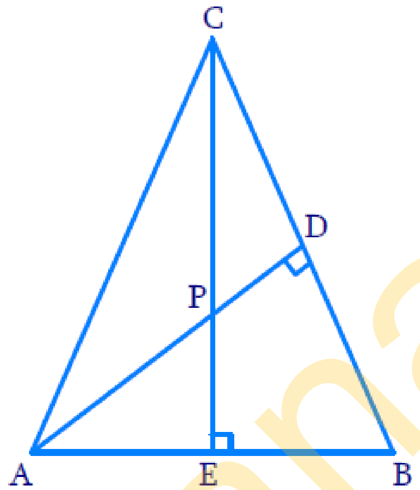
$(i)\ \Delta AEP \sim \Delta CDP$

$(ii)\ \Delta ABD \sim \Delta CBE$

$(iii)\ \Delta AEP \sim \Delta ADB$

$(iv)\ \Delta PDC \sim \Delta BEC$

Diagram



(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution:

In ΔAEP and ΔCDP

$$\angle AEP = \angle CDP = 90^\circ$$

$$[\therefore CE \perp AB \text{ and } AD \perp BC; \text{ altitudes}]$$

$$\angle APE = \angle CPD \ (\text{Vertically opposite angles})$$

$$\Rightarrow \Delta AEP \sim \Delta CPD (\text{AA criterion})$$

(ii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

In ΔABD and ΔCBE

$$\angle ADB = \angle CEB = 90^\circ$$

$$\angle ABD = \angle CBE \ (\text{Common angle})$$

$$\Rightarrow \Delta ABD \sim \Delta CBE (\text{AA criterion})$$

(iii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

In ΔAEP and ΔADB

$$\angle AEP = \angle ADB = 90^\circ$$
$$\angle PAE = \angle BAD \text{ (Common angle)}$$
$$\Rightarrow \Delta AEP \sim \Delta ADB$$

(iv) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred as AA criterion for two triangles.

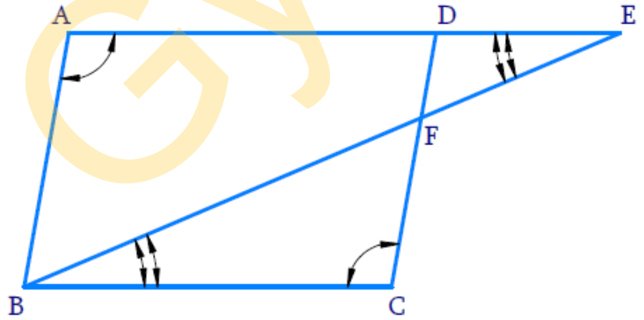
Solution

In ΔPDC and ΔBEC

$$\angle PDC = \angle BEC = 90^\circ$$
$$\angle PCD = \angle BCE \text{ (Common angle)}$$
$$\Rightarrow \Delta PDC \sim \Delta BEC$$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta\ ABE \sim \Delta CFB$.

Diagram



Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

In ΔABE and ΔCFB

$\angle BAE = \angle FCB$ (opposite angles of a parallelogram)

$\angle AEB = \angle FBC$ [$\because AE \parallel BC$ and EB is a transversal, alternate interior angle]

$\Rightarrow \Delta ABE \sim \Delta CFE$ (AA criterion)

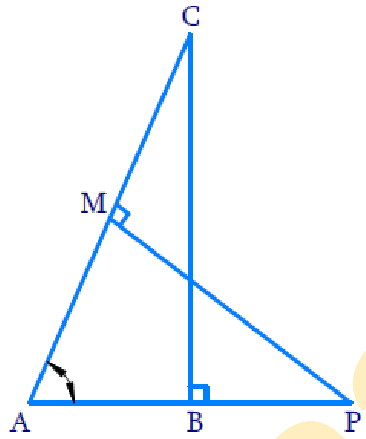
Q9. In Figure 6.39, ABC and AMP are two right triangles, right angled at B and M respectively.

Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Diagram



(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

In ΔABC and ΔAMP

$$\angle ABC = \angle AMP = 90^\circ$$
$$\angle BAC = \angle MAP \text{ (Common angle)}$$
$$\Rightarrow \Delta ABC \sim \Delta AMP$$

(ii) Reasoning:

As we know that the ratio of any two corresponding sides in two equiangular triangles is always the same

Solution

In ΔABC and ΔAMP

$$\frac{CA}{PA} = \frac{BC}{MP} \quad [\because \Delta ABC \sim \Delta AMP]$$

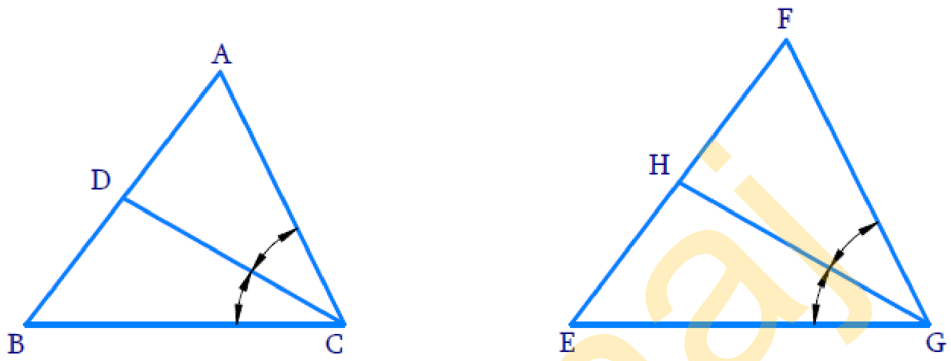
Q10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Diagram



(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

$\angle ACB = \angle FGE$

$\Rightarrow \frac{\angle ACB}{2} = \frac{\angle FGE}{2}$

$\Rightarrow \angle ACD = \angle FGH$ (CD and GH are bisectors of $\angle C$ and $\angle$G respectively)

In ΔADC and ΔFHG

$\angle DAC = \angle HFG$ $[\because \Delta ADC \sim \Delta FEG]$

$\angle ACD = \angle FGH$

$\Rightarrow \Delta ADC \sim \Delta FHG$ (AA criterion)

[If two triangles are similar, then their corresponding sides are in the same ratio]

$\Rightarrow \frac{CD}{GH} = \frac{AG}{FG}$

(ii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This is referred as AA criterion for two triangles.

Solution

In ΔDCB and ΔHGE

$$\angle DBC = \angle HEG \quad [\because \Delta ABC \sim \Delta FEG]$$

$$\angle DCB = \angle HGE \quad \left[\because \frac{\angle ACB}{2} = \frac{\angle FGE}{2}\right]$$

$$\Rightarrow \Delta DCB \sim \Delta EHG \quad \text{(AA criterion)}$$

(iii) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

Solution

In ΔDCA, ΔHGF

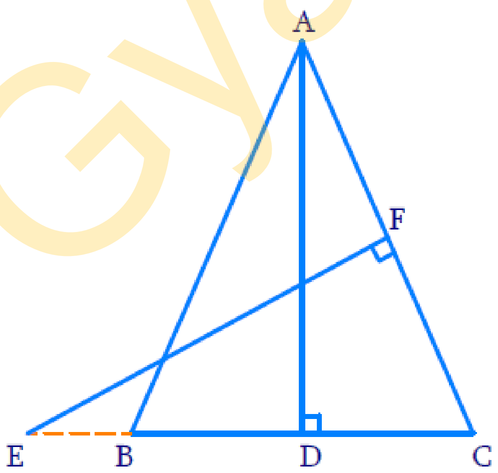
$$\angle DAC = \angle HFG \quad [\because \Delta ABC \sim \Delta FEG]$$

$$\angle ACD = \angle FGH \quad \left[\because \frac{\angle ACB}{2} = \frac{\angle FGE}{2}\right]$$

$$\Rightarrow \Delta DCA \sim \Delta HGF \quad \text{(AA criterion)}$$

Q11. In Figure 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.

Diagram



Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This is referred as AA criterion for two triangles.

Solution

In ΔABD and ΔECF

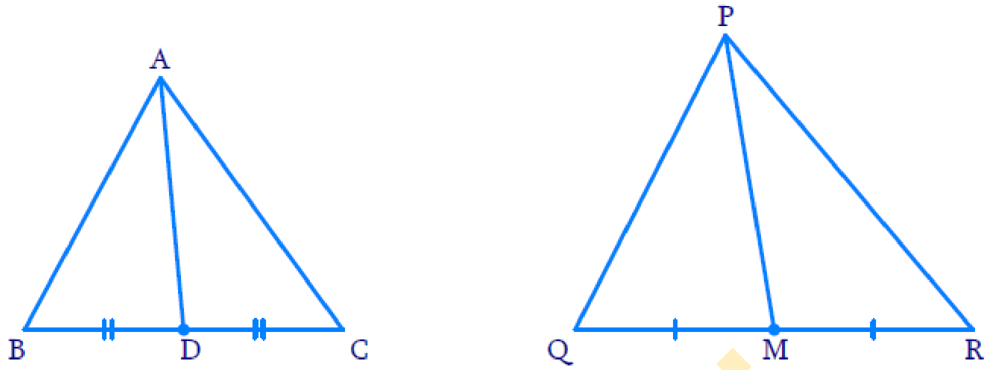
$$\angle ADB = \angle EFC = 90^\circ \quad [\because AD \perp BC \text{ and } EF \perp AC]$$

$$\angle ABD = \angle ECF \quad [\because \text{In } \Delta ABC, AB = AC \Rightarrow \angle ABC = \angle ACB]$$

$$\Rightarrow \Delta ABD \sim \Delta ECF \quad \text{(AA criterion)}$$

Q12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ, QR and median PM of Δ PQR (see Figure 6.41). Show that Δ ABC ~ Δ PQR.

Diagram



Reasoning:

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This is referred as SAS criterion for two triangles.

Solution

In ΔABC and ΔPQR

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM} \quad [\text{given}]$$

AD and PM are median of ΔABC and ΔPQR respectively

$$\Rightarrow \frac{BD}{QM}=\frac{BC/2}{QR/2}=\frac{BC}{QR}$$

Now In ΔABD and ΔPQM

$$\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$$

$$\Rightarrow \Delta ABD \sim \Delta PQM$$

Now In ΔABC and ΔPQR

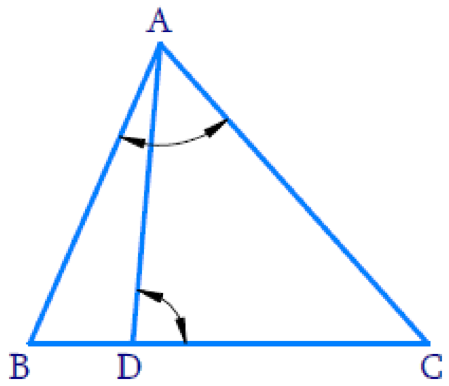
$$\frac{AB}{PQ}=\frac{BC}{QR} \quad [\text{given in the statement}]$$

$$\angle ABC = \angle PQR \quad [\because \Delta ABD \sim \Delta PQM]$$

$$\Rightarrow \Delta ABC \sim \Delta PQR \quad [\text{SAS criteion}]$$

Q13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠ BAC. Show that $CA^2 = CB.CD$.

Diagram



Reasoning:

As we know that if two triangles are similar, then their corresponding sides are proportional.

Solution

In ΔABC and ΔDAC

$$\angle BAC = \angle ADC \quad \text{(Given in the statement)}$$

$$\angle ACB = \angle ACD \quad \text{(Common angles)}$$

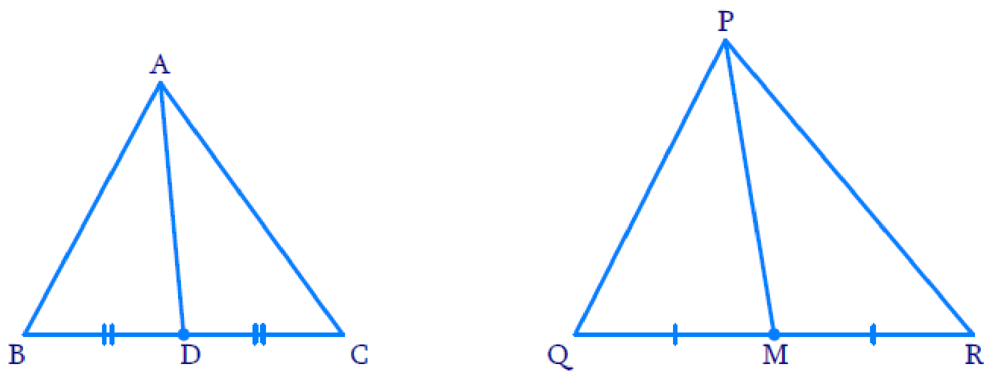
$$\Rightarrow \Delta ABC \sim \Delta DAC \quad \text{(AA criterion)}$$

If two triangles are similar, then their corresponding sides are proportional

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$$

$$\Rightarrow CA^2 = CB \cdot CD$$

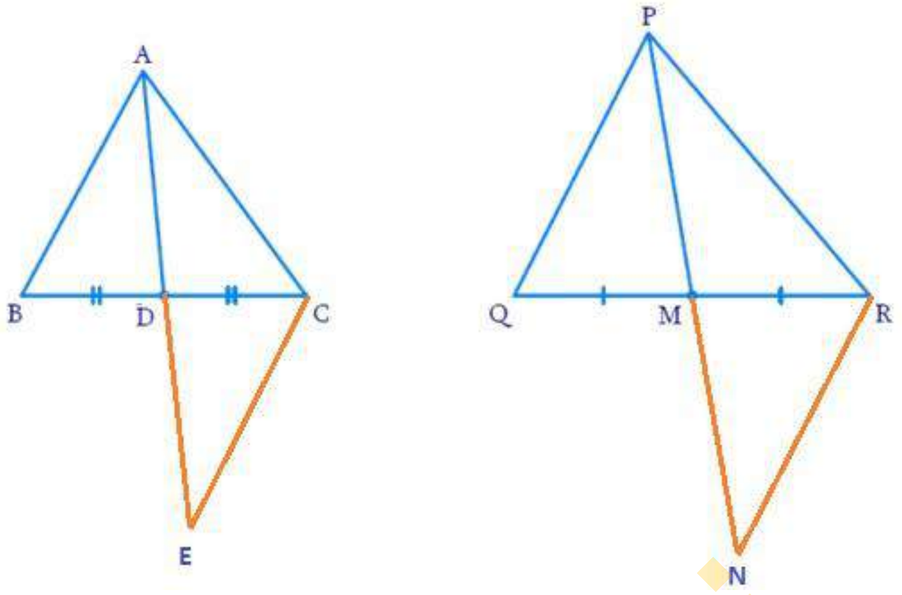
Q14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.



Reasoning:

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This is referred as SAS criterion for two triangles.

Solution



Produce AD to E so that $AD = DE$. Join CE

Similarly, produce PM to N such that $PM = MN$, and Join RN.

In ΔABD and ΔCDE

$AD = DE$ [By Construction]

$BD = DC$ [$\because$ AP is the median]

$\angle ADB = \angle CDE$ [Vertically opposite angles]

$\therefore \Delta ABD \cong \Delta CDE$ [By SAS criterion of congruence]

$\Rightarrow AB = CE$ [CPCT] ...(i)

Also, in ΔPQM and ΔMNR

$PM = MN$ [By Construction]

$QM = MR$ [$\because$ PM is the median]

$\angle PMQ = \angle NMR$ [Vertically opposite angles]

$\therefore \Delta PQM = \Delta MNR$ [By SAS criterion of congruence]

$\Rightarrow PQ = RN$ [CPCT] ...(ii)

Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ [Given]

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$ [from (i) and (ii)]

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [$\because 2AD = AE$ and $2PM = PN$]

$\therefore \Delta ACE \sim \Delta PRN$ [By SSS similarity criterion]

Therefore, $\angle CAE = \angle RPN$

Similarly, $\angle BAE = \angle QPN$

$\therefore \angle CAE + \angle BAE = \angle RPN + \angle QPN$

$\Rightarrow \angle BAC = \angle QPR$

$\Rightarrow \angle A = \angle P$ ...(iii)

Now, In ΔABC and ΔPQR

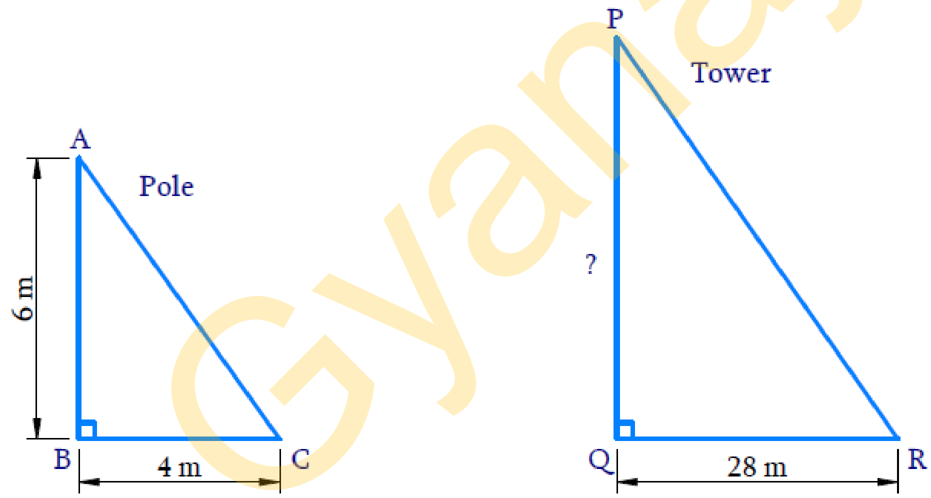
$\frac{AB}{PQ} = \frac{AC}{PR}$

$\angle A = \angle P$ [from (iii)]

$\therefore \Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

Q15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Diagram



Reasoning:

The ratio of any two corresponding sides in two equiangular triangles is always the same.

Solution

AB is the pole = 6m

BC is the shadow of pole = 4m

PQ is the tower =?

QR is the shadow of the tower = 28m

In ΔABC and ΔPQR

$\angle ABC = \angle PQR = 90^\circ$ (The objects and shadows are perpendicular to each other)

$\angle BAC = \angle QPR$ (Sunray fall on the pole and tower at the same angle, at the same time)

$\Rightarrow \Delta ABC \sim \Delta PQR$ (AA criterion)

The ratio of any two corresponding sides in two equiangular triangles is always the same.

$$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$$

$$\Rightarrow \frac{6m}{4m} = \frac{PQ}{28m}$$

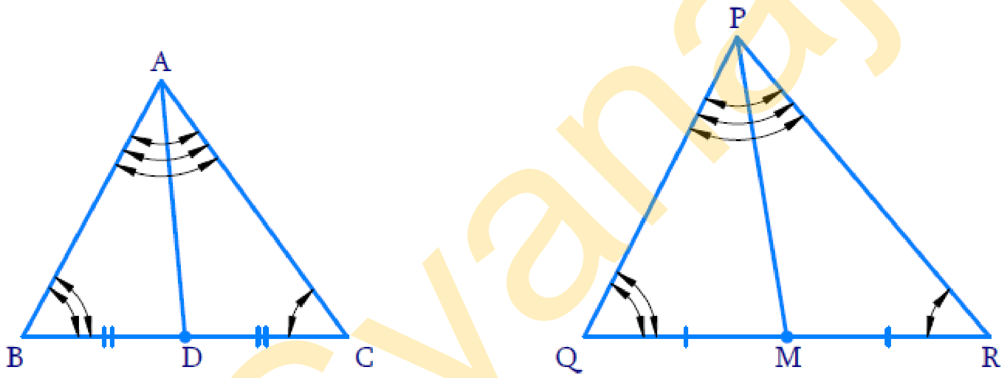
$$\Rightarrow PQ = \frac{6 \times 28}{4} m$$

$$\Rightarrow PQ = 42m$$

Hence, the height of the tower is 42m.

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Diagram



Reasoning:

As we know If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This is referred as SAS criterion for two triangles.

Solution

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \angle ABC = \angle PQR \quad \text{(corresponding angles)} \quad (1)$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \quad \text{(corresponding sides)}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC/2}{QR/2}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad \text{(D and M are mid-points of BC and QR)} \quad (2)$$

In ΔABD and ΔPQM

$$\angle ABD = \angle PQM \qquad (\text{from } 1)$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \qquad (\text{from } 2)$$

$$\Rightarrow \Delta ABD \sim \Delta PQM \qquad (\text{SAS criterion})$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \qquad (\text{corresponding sides})$$

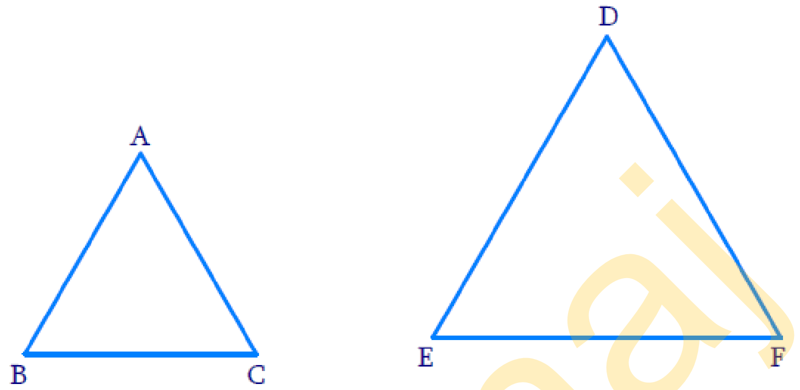
$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

Chapter - 6: Triangles

Exercise 6.4 (Page 143)

Q1. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF =15.4 cm, find BC.

Diagram



Reasoning:

As we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution

$$\Delta ABC \sim \Delta DEF$$

$$\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{(BC)^2}{(EF)^2}$$

$$\frac{64cm^2}{121cm^2} = \frac{(BC)^2}{(15.4)^2}$$

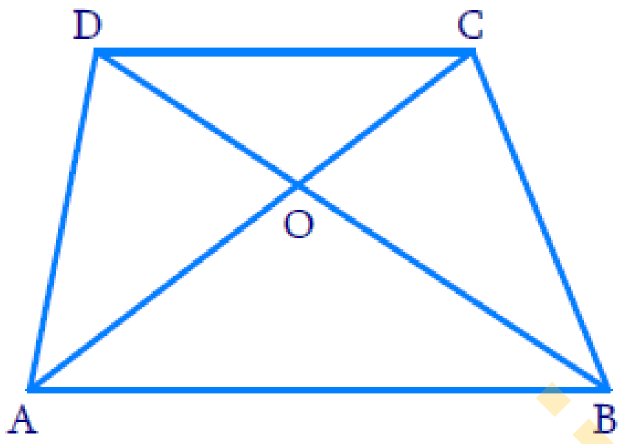
$$(BC)^2 = \frac{(15.4)^2 \times 64}{121}$$

$$BC = \frac{15.4 \times 8}{11}$$

$$BC = 11.2 \text{ cm}$$

Q2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Diagram



Reasoning:

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

AA criterion.

Solution

In trapezium ABCD, $AB \parallel CD$ and $AB = 2CD$

Diagonals AC, BD intersect at 'O'

In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ (vertically opposite angles)

$\angle ABO = \angle CDO$ [alternate interior angles]

$\Rightarrow \Delta AOB \sim \Delta COD$ (AA criterion)

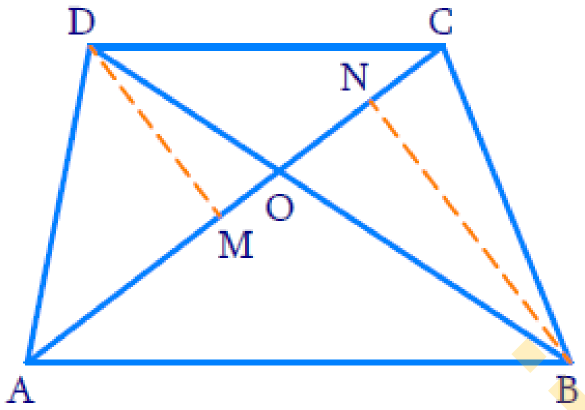
$$\Rightarrow \frac{Area\,of\,\Delta AOB}{Area\,of\,\Delta COD} = \frac{(AB)^2}{(CD)^2} \quad \text{[theorem 6.6]}$$

$$= \frac{(2CD)^2}{(CD)^2} = \frac{4CD^2}{CD^2} = \frac{4}{1}$$

$\Rightarrow$ Area of ΔAOB :area of ΔCOD =4:1

Q3. In Fig. 6.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{area(ABC)}{area(DBC)} = \frac{AO}{DO}$

Diagram



Reasoning:

AA criterion

Solution:

In ΔABC

Draw $AM \perp BC$

In ΔDBC

Draw $DN \perp BC$

Now in $\Delta AOM, \Delta DON$

$\angle AMO = \angle DNO = 90^\circ$

$\angle AOM = \angle DON$ (Vertically opposite angles)

$\Rightarrow \Delta AOM \sim \Delta DON$ (AA criterion)

$\Rightarrow \frac{AM}{DN} = \frac{OM}{ON} = \frac{AO}{DO}$ ………… (1)

Now,

$$Area\ of\ \Delta ABC = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times BC \times AM$$

$$Area\ of\ \Delta DBC = \frac{1}{2} \times BC \times DN$$

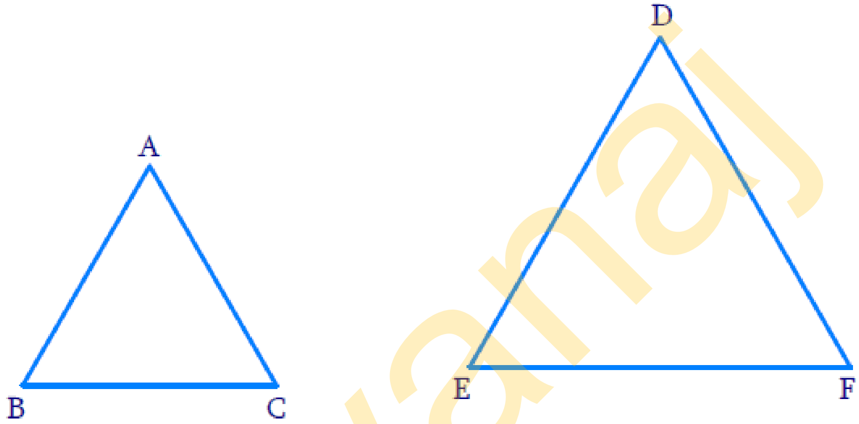
$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC}=\frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times BC\times DN}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC}=\frac{AM}{DN}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC}=\frac{AO}{DO}\ (\text{from}(1))$$

Q4. If the areas of two similar triangles are equal, prove that they are congruent.

Diagram



Reasoning:

As we know that two triangular are similar if their corresponding angles are equal and their corresponding sides are in the same ratio. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

As we know if three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

Solution:

$$\Delta ABC \sim \Delta DEF$$

$$\Rightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}\ (\text{SSS criterion})$$

But area of ΔABC = area of ΔDEF

$$\Rightarrow \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF}=1 \quad ....(1)$$

But $$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF}=\frac{(AB)^2}{(DE)^2}=\frac{(BC)^2}{(EF)^2}=\frac{(CA)^2}{(FD)^2}$$

From (1)

$$\frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(CA)^2}{(FD)^2} = 1$$

$$\Rightarrow \frac{(AB)^2}{(DE)^2} = 1$$

$$\Rightarrow (AB)^2 = (DE)^2$$

$$\Rightarrow AB = DE \text{......................}(2)$$

Similarly,

$$\Rightarrow BC = EF \text{ ....}(3)$$

$$\Rightarrow CA = FD \text{...............}(4)$$

Now, In ΔABC and ΔDEF

$$\Rightarrow AB = DE \qquad (\text{from } 2)$$

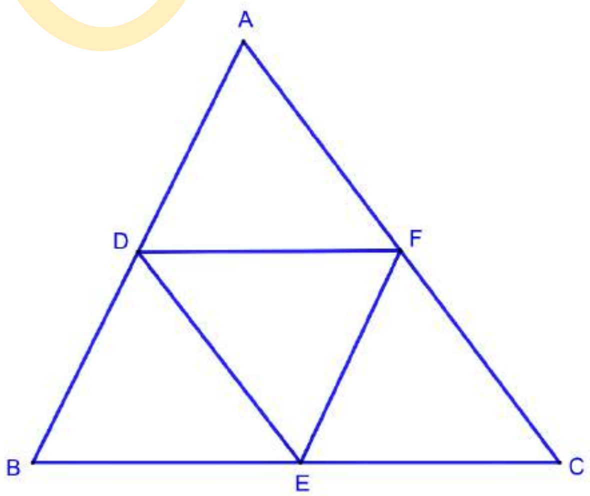
$$\Rightarrow BC = EF \qquad (\text{from } 3)$$

$$\Rightarrow CA = FD \qquad (\text{from } 4)$$

$$\Rightarrow \Delta ABC \cong \Delta DEF \text{ (SSS congruency)}$$

Q5. D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

Diagram



Reasoning:

As we know that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it – (mid-point theorem).

Solution

In ΔABC, D and F are mid-points of AB and AC respectively.

$\Rightarrow DF \parallel BC$ and $DF = \frac{1}{2}BC$ (by mid-point theorem)

Again, E is the mid-point of BC

$\Rightarrow DF \parallel BE$ and $DF = BE$

In quadrilateral $DFEB$

$DF \parallel BE$ and $DF = BE$

$\therefore DFEB$ is a parallelogram

$\Rightarrow \angle B = \angle F$ (1) (opposite angles of a parallelogram are equal)

Similarly, we can prove that,

$DFCE$ is a parallelogram

$\Rightarrow \angle C = \angle D$ (2) (opposite angles of a parallelogram are equal)

Now, In ΔDEF and ΔABC

$\angle DFE = \angle ABC$ (from 1)

$\angle EDF = \angle ACB$ (from 2)

$\Rightarrow \Delta DEF \sim \Delta CAB$ (AA criterion)

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{Area\,of\ \Delta DEF}{Area\,of\ \Delta ABC} = \frac{(DE)^2}{(AC)^2} = \frac{(EF)^2}{(AB)^2} = \frac{(DF)^2}{(BC)^2}$$

$$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta ABC} = \frac{(DF)^2}{(BC)^2}$$

$$= \frac{(\frac{1}{2}BC)^2}{(BC)^2}$$

$$= \frac{BC^2}{4BC^2}$$

$$= \frac{1}{4}$$

The ratio of the areas of ΔDEF and ΔABC is 1:4

Alternate method:

Reasoning:

Theorem 8.9 midpoint theorem Pg. No.148.

Solution:

In ΔABC D and E are midpoints of sides AB and AC

$\Rightarrow DE \| BC$ and $DE = \frac{1}{2}BC$ ....(1)

Now in quadrilateral DBFE

$\Rightarrow DE \| BC$ and $DE = BF$ (from 1)

$\Rightarrow$ DBFE is a parallelogram

$\Rightarrow$ Area of ΔDBF = area of ΔDEF ....(2)

($\because$ diagonal DF divides the parallelogram into two triangle of equal area)

Similarly, we can prove

Area of ΔDBF = Area of ΔEFC ....(3)

And area of ΔDEF = Area of ΔADE ....(4)

From (2) (3) and (4)

Area of ΔDBF = Area of ΔDEF = Area of ΔEFC =Area of ΔADE ....(5)

(Things which are equal to the same thing are equal to one another – Euclid's 1st axiom.)

Area of ΔABC = Area of ΔADE + Area of DBF+ Area of ΔEFD +Area of ΔDEF

From (5)

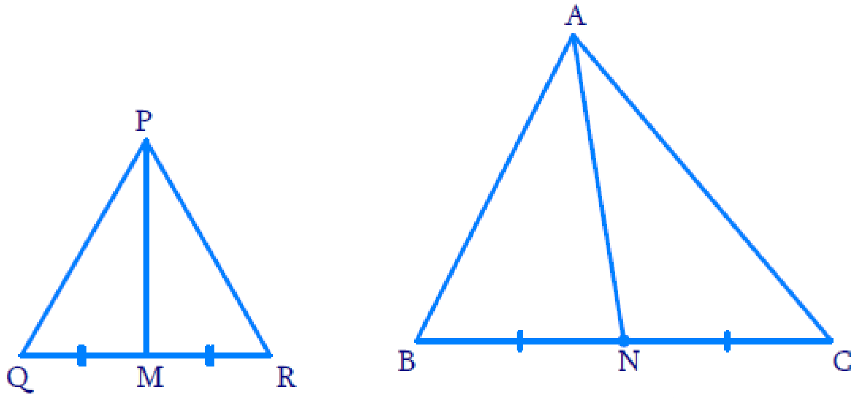
Area of $\Delta ABC = 4 \times$ Area of ΔDEF

$$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta ABC} = \frac{1}{4}$$

Area of ΔDEF : Area of ΔABC = 1:4

Q6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Diagram



Reasoning:

As we know, if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. And we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:

In ΔPQR ,PM is the median and, In ΔABC AN is the median

$$\Delta PQR \sim \Delta ABC \text{ (given)}$$

$$\angle PQR = \angle ABC \text{................}(1)$$

$$\angle QPR = \angle BAC \text{ ....}(2)$$

$$\angle QRP = \angle BCA \text{ ....}(3)$$

$$\text{and } \frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA} \text{ .... } (4)$$

($\because$ If two triangles are similar, then their corresponding angles are equal and corresponding sides are in the same ratio)

$$\frac{Area\,of\ \Delta PQR}{Area\,of\ \Delta ABC} = \frac{(PQ)^2}{(AB)^2} = \frac{(QR)^2}{(BC)^2} = \frac{(RP)^2}{(CA)^2} \text{ [THEROM 6.6] .... } (5)$$

Now In ΔPQM and ΔABN

$$\angle PQM = \angle ABN \text{ (from 1)}$$

$$\text{And } \frac{PQ}{AB} = \frac{QM}{BN}$$

$$\left[\because \frac{PQ}{AB}=\frac{QR}{BC}=\frac{2QM}{2BN}; M,\ N \text{ mid points of } QR \text{ and } BC\right]$$

$\Rightarrow \Delta PQM \sim \Delta ABN$ [SAS similarly]

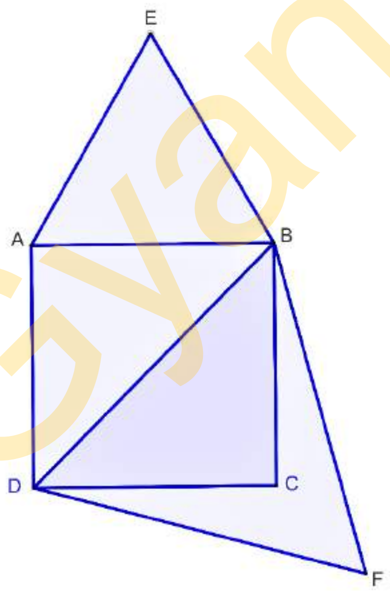
$$\Rightarrow \frac{Area\, of\ \Delta PQM}{Area\, of\ \Delta ABN}=\frac{(PQ)^2}{(AB)^2}=\frac{(QM)^2}{(BN)^2}=\frac{(PM)^2}{(AN)^2}\left[\because theorem\ 6.6\right] \text{ .... } (6)$$

From (5) and (6)

$$\frac{Area\, of\ \Delta PQR}{Area\, of\ \Delta ABC}=\frac{(PM)^2}{(AN)^2}$$

Q7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Diagram



Reasoning:

As we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:

ΔABE is described on the side AB of the square ABCD

ΔDBF is described on the diagonal BD of the square ABCD

Since ΔABE and ΔDBF **are equilateral triangles**

$$\Delta ABE \sim \Delta DBF \quad \left[\text{each angle in equilateral triangles is } 60^\circ\right]$$

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{(AB)^2}{(DB)^2}$$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{(AB)^2}{\left(\sqrt{2}AB\right)^2} \qquad [\text{diagonal of a square is } \sqrt{2} \times side]$$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{AB^2}{2AB^2}$$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{1}{2}$$

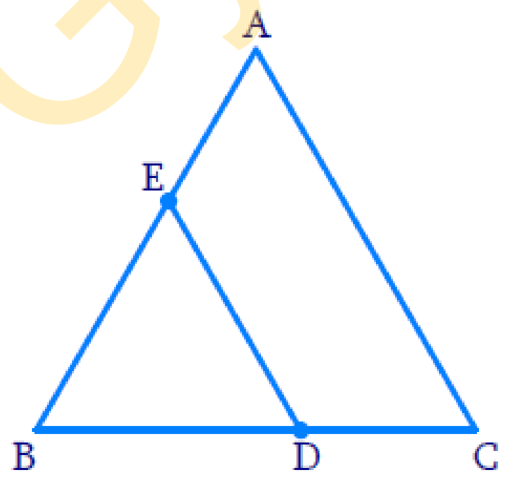
$$Area\ of\ \Delta ABE = \frac{1}{2} \times Area\ of\ \Delta DBF$$

Tick the correct answer and justify:

Q8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

Diagram



Reasoning:
AAA criterion.

Solution:

$$\Delta ABC \sim \Delta BDE \ (\because \text{ equilateral triangles})$$

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

$$\frac{Area\ \Delta ABC}{Area\ \Delta BDE} = \frac{(BC)^2}{(BD)^2}$$

$$= \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2} \qquad (\text{ D is the midpoint of BC})$$

$$= \frac{(BC)^2 \times 4}{(BC)^2}$$

$$= 4$$

$$Area\ \Delta ABC : Area\ \Delta BDE = 4:1$$

Answer (c)
4:1

Q9. Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio

(A) 2:3 (B) 4:9 (C) 81:16 (D) 16:81

Reasoning:
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

Solution
We know that,
Ratio of the areas of two similar triangles = square of the ratio of their corresponding sides

$$= (4:9)^2$$
$$= 16:81$$

Answer (d)
16:81

Chapter - 6: Triangles

Exercise 6.5(Page 150)

Q1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Reasoning:
As we know, in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Solution
(i) $(25)^2 = 625$

$$\begin{aligned} 7^2 + (24)^2 &= 49 + 576 \\ &= 625 \\ \therefore\ (25)^2 &= 7^2 + (24)^2 \end{aligned}$$

Length of hypotenuse = 25cm

(ii) $8^2 = 64$

$$\begin{aligned} 3^2 + 6^2 &= 9 + 36 \\ &= 45 \\ 8^2 &\neq 3^2 + 6^2 \end{aligned}$$

(iii) $(100)^2 = 10000$

$$\begin{aligned} (50)^2 + (80)^2 &= 2500 + 6400 \\ &= 8900 \\ (100)^2 &\neq (50)^2 + (80)^2 \end{aligned}$$

(iv) $(13)^2 = 169$

$$(12)^2 + 5^2 = 144 + 25$$
$$= 169$$

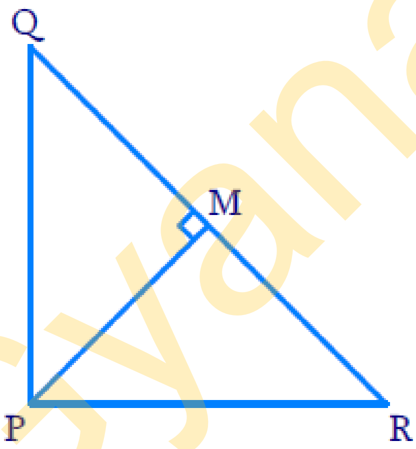
$$\therefore (13)^2 = (12)^2 + 5^2$$

Length of hypotenuse = 13cm

$\Rightarrow$ (i) and (iv) are right triangle.

Q2. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $(PM)^2$ = QM. MR

Diagram



Reasoning:
As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Solution
In ΔPQR; $\angle QPR = 90^0$ and $PM \perp QR$

In ΔPQR and ΔMQP

$$\angle QPR = \angle QMP = 90^0$$
$$\angle PRQ = \angle MQP \quad \text{(commom angle)}$$
$$\Rightarrow \Delta PQR \sim \Delta MQP \quad \text{(AA Similarity)} \quad (1)$$

In ΔPQR and ΔMPR

$$\angle QPR = \angle PMR = 90^0$$

$$\angle PRQ = \angle PRM \qquad (\text{commom angle})$$

$$\Rightarrow \Delta PQR \sim \Delta MPR \qquad (\text{AA Similarity}) \qquad (2)$$

From (1) and (2)

$$\Delta MQP \sim \Delta MPR$$

$$\frac{PM}{MR} = \frac{QM}{PM} \qquad (\text{corresponding sides of similar traingles are proportional})$$

$$\Rightarrow PM^2 = QM.MR$$

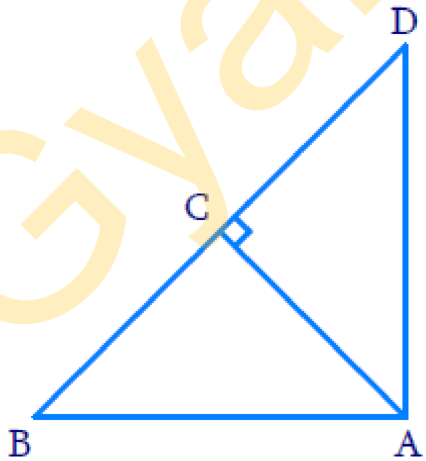
Q3. In Fig. 6.53, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$

Diagram



Reasoning:

As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Solution:

i). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$$\Rightarrow \Delta BAD \sim \Delta BCA$$

$$\Rightarrow \frac{AB}{BC} = \frac{BD}{AB} \qquad (\text{Corresponding sides of similar triangle})$$

$$\Rightarrow AB^2 = BC \cdot BD$$

ii). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$$\Rightarrow \Delta BCA \sim \Delta ACD$$

$$\Rightarrow \frac{AC}{CD} = \frac{BC}{AC} \quad \text{(Corresponding sides of similar triangle)}$$

$$\Rightarrow AC^2 = BC \cdot DC$$

iii). As we know if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

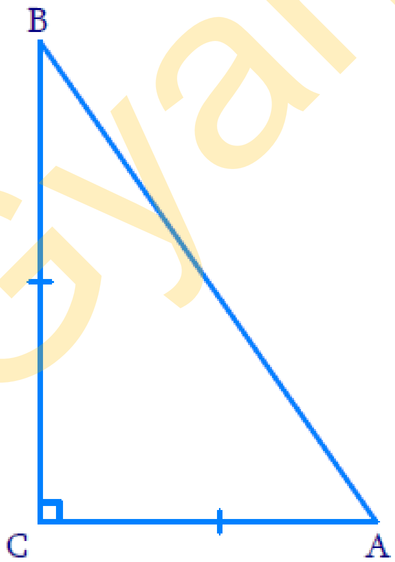
$$\Rightarrow \Delta BAD \sim \Delta ACD$$

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD} \quad \text{(Corresponding sides of similar triangle)}$$

$$AD^2 = BD \cdot CD$$

Q4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Diagram



Reasoning:

As we are aware, in a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

Solution:

In ΔABC, $\angle ACB = 90^0$ and $AC = BC$

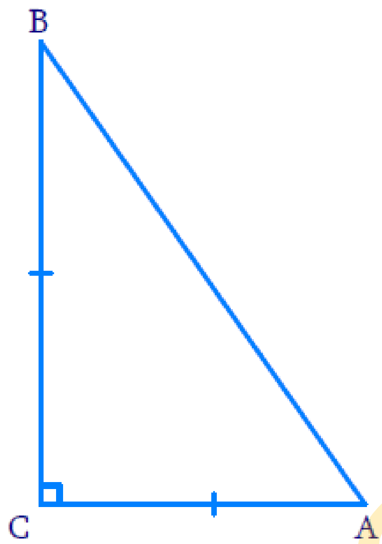
$$\text{But } AB^2 = AC^2 + BC^2$$

$$= AC^2 + AC^2 [\because AC = BC]$$

$$AB^2 = 2AC^2$$

Q5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Diagram



Reasoning:

As we know, in a triangle, if square of one side is equal to the sum of the square of the other two sides then the angle opposite the first side is a right angle.

Solution

In ΔABC

$$AC = BC$$

And $AB^2 = 2AC^2$

$$= AC^2 + AC^2$$

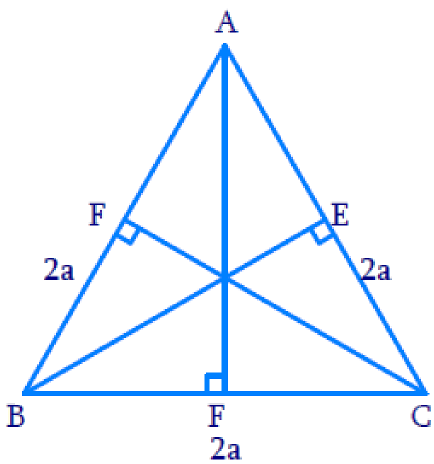
$$AB^2 = AC^2 + BC^2 \; [\because AC = BC]$$

$\Rightarrow \angle ACB = 90^\circ$

$\Rightarrow \Delta ABC$ is a right triangle

Q6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Diagram



Reasoning:

We know that in an equilateral triangle perpendicular drawn from vertex to the opposite side, bisects the side.

As we know that, in a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides

Solution

In ΔABC

$$AB = BC = CA = 2a$$
$$AD \perp BC$$
$$\Rightarrow BD = CD = \frac{1}{2} BC = a$$

In ΔADB

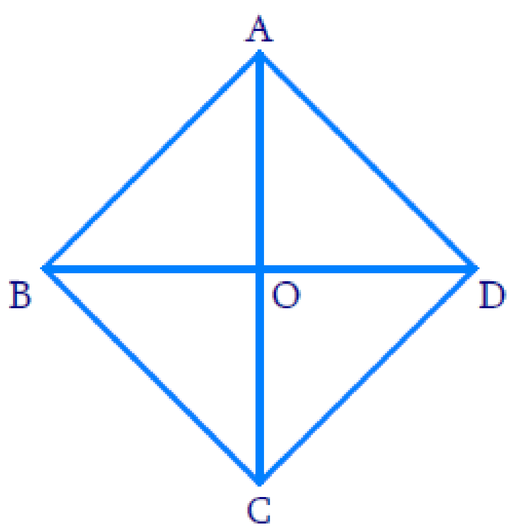
$$AB^2 = AD^2 + BD^2$$
$$AD^2 = AB^2 - BD^2$$
$$= (2a)^2 - a^2$$
$$= 4a^2 - a^2$$
$$= 3a^2$$
$$AD = \sqrt{3}a$$

$$\Rightarrow AD = \sqrt{3}a \text{ units}$$

Similarly, we can prove that, $BE = CF = \sqrt{3}a$ units

Q7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Diagram



Reasoning:
As we know, in a rhombus, diagonals bisect each other perpendicularly.

Solution:
In rhombus ABCD

$AC \perp BD$ and OA = OC; OB = OD

In ΔAOB

$$\angle AOB = 90^0$$
$$\Rightarrow AB^2 = OA^2 + OB^2 \ldots\ldots\ldots (1)$$

Similarly, we can prove

$$BC^2 = OB^2 + OC^2 \ldots\ldots\ldots\ldots(2)$$
$$CD^2 = OC^2 + OD^2 \ldots\ldots\ldots\ldots(3)$$
$$AD^2 = OD^2 + OA^2 \ldots\ldots\ldots\ldots(4)$$

Adding (1), (2), (3) and (4)

$$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$$

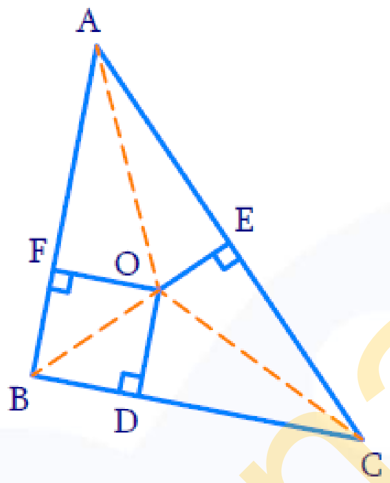
$$AB^2 + BC^2 + CD^2 + AD^2 = 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$
$$AB^2 + BC^2 + CD^2 + AD^2 = 2[OA^2 + OB^2 + OC^2 + OD^2]$$
$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$$
$$\left[\because OA = OC = \frac{AC}{2} \text{ and } OB = OD = \frac{BD}{2}\right]$$
$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{AC^2 + BD^2 + AC^2 + BD^2}{4}\right]$$
$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\frac{2AC^2 + 2BD^2}{4}\right]$$
$$AB^2 + BC^2 + CD^2 + AD^2 = 4\left[\frac{AC^2 + BD^2}{4}\right]$$
$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Q8. In Figure 6.54, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥AB. Show that

i. $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

ii. $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

(i) In ΔABC

$$OD \perp BC, OE \perp AC \text{ and } OF \perp AB$$

OA, OB and OC joined

In ΔOAF

$$OA^2 = AF^2 + OF^2 [\because \angle OFA = 90^0] \ldots\ldots\ldots\ldots (1)$$

Similarly, In ΔOBD

$$OB^2 = BD^2 + OD^2 [\because \angle ODA = 90^0] \ldots\ldots\ldots\ldots (2)$$

In ΔOCE

$$OC^2 = CE^2 + OE^2 \left[\because \angle OEC = 90^\circ\right] \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Adding (1), (2) and (3)

$$OA^2 + OB^2 + OC^2 = AF^2 + OF^2 + BD^2 + OD^2 + CE^2 + OE^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \ldots\ldots\ldots\ldots (4)$$

(ii) From (4)

$$(OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$$

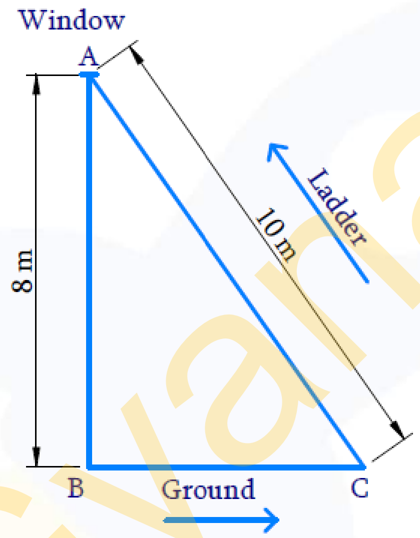
(Rearranging the left side terms)

$$AE^2 + BF^2 + CD^2 = AF^2 + BD^2 + CE^2$$

[$\because$ $\Delta OAE, \Delta OBD$ and ΔOCE are right triangles]

Q9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

AB is height of the windows from the ground = 8m

AC is the length of the ladder = 10m

BC is the foot of the ladder from the base of ground = ?

Since ΔABC is right angled triangle $(\angle ABC = 90^0)$

$$BC^2 = AC^2 - AB^2 \quad \text{(Pythagoras theorem)}$$

$$BC^2 = 10^2 - 8^2$$

$$BC^2 = 100 - 64$$

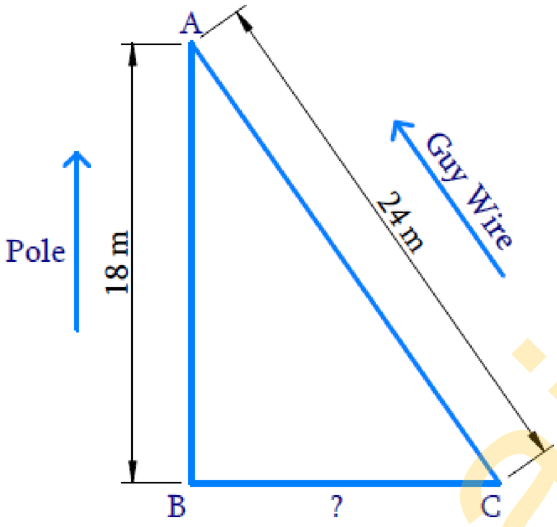
$$BC^2 = 36$$

$$BC = 6\ m$$

The distance of the foot of the ladder from the base of the wall is 6m

Q10. A guy wire attached to a vertical pole of height 18m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution

AB is the length of the pole = 18m

AC is the length of the guy wire = 24m

BC is the distance of the stake from the pole = ?

In $\Delta ABC \quad \angle ABC = 90^0$

$$BC^2 = AC^2 - AB^2 \quad \text{(Pythagoras theorem)}$$

$$BC^2 = 24^2 - 18^2$$

$$BC^2 = 576 - 324$$

$$BC^2 = 252$$

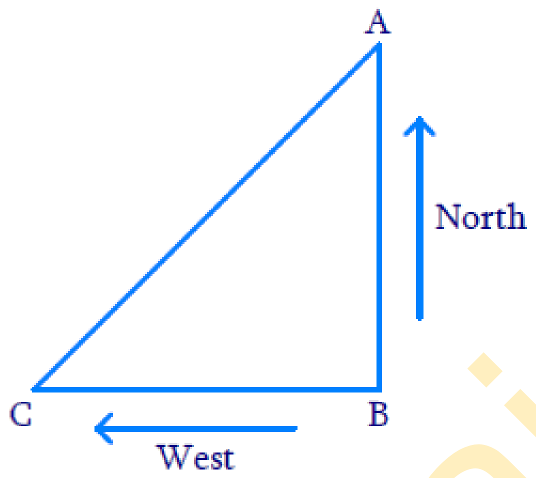
$$BC = 2 \times 3\sqrt{7}$$

$$BC = 6\sqrt{7}$$

The distance of the stake from the pole is $6\sqrt{7}m$

Q11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Diagram



Reasoning:

We have to find the distance travelled by aeroplanes, we need to use

$$\text{distance} = \text{speed} \times \text{time}$$

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

AB is the distance travelled by aeroplane travelling towards north

$$AB = 1000\,km/hr \times 1\frac{1}{2}hr$$

$$= 1000 \times \frac{3}{2}\,km$$

$$AB = 1500\ km$$

BC is the distance travelled by another aeroplane travelling towards south

$$BC = 1200\ km/hr \times 1\frac{1}{2}\ hr$$

$$= 1200 \times \frac{3}{2}hr$$

$$BC = 1800\ km$$

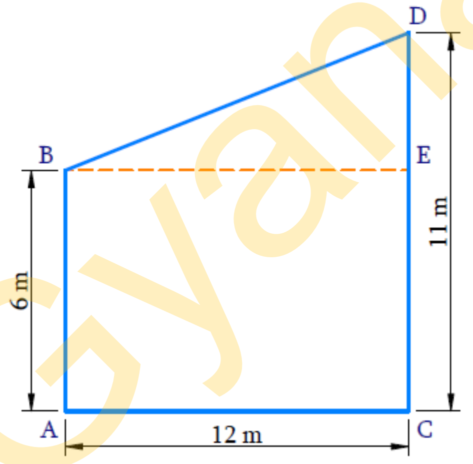
Now, In ΔABC , $\angle ABC = 90^0$

$$AC^2 = AB^2 + BC^2 \text{ (Pythagoras theorem)}$$
$$= (1500)^2 + (1800)^2$$
$$= 2250000 + 3240000$$
$$AC^2 = 5490000$$
$$AC = \sqrt{549000}$$
$$= 300\sqrt{61} \text{ km}$$

The distance between two planes after $1\frac{1}{2} hr = 300\sqrt{61}\ km$

Q12. Two poles of heights 6 m and 11 m stand on plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

AB is the height of one pole = 6m

CD is the height of another pole = 11m

AC is the distance between two poles at bottom = 12m

BD is the distance between the tops of the poles =?

Draw $BE \parallel AC$

Now consider, In ΔBED

$$\angle BED = 90^0$$

BE = AC = 12 m

DE = CD – CE

DE = 11 – 6 = 5 cm

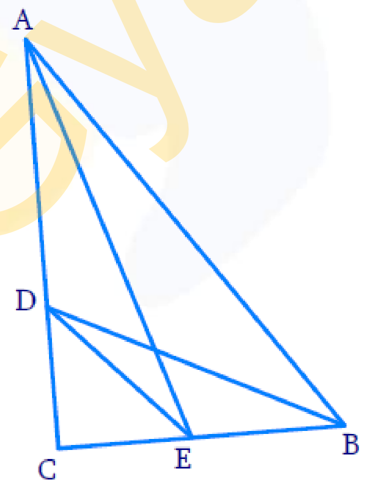
Now

$$BD^2 = BE^2 + DE^2 \quad \text{(Pythagoras therorem)}$$
$$= 12^2 + 5^2$$
$$= 144 + 25$$
$$BD^2 = 169$$
$$BD = 13m$$

The distance between the tops of poles =13m

Q13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution

In ΔABC, $\angle ACB = 90^0$

D, E are points on AC, BC

Join AE, DE and BD

In ΔACE

$$AE^2 = AC^2 + CE^2 \text{ (Pythagoras theorem) ………. (1)}$$

In ΔDCB

$$BD^2 = CD^2 + BC^2 \text{ …………….. (2)}$$

Adding (1) and (2)

$$\begin{aligned} AE^2 + BD^2 &= AC^2 + CE^2 + CD^2 + BC^2 \\ &= AC^2 + BC^2 + EC^2 + CD^2 \\ &= AB^2 + DE^2 \end{aligned}$$

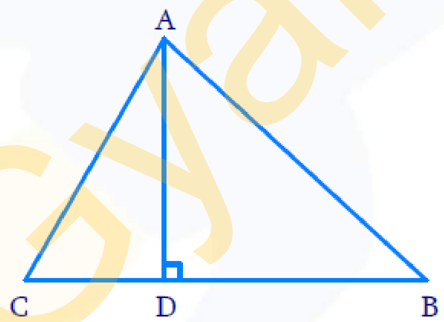
$[In\ \Delta ABC, \quad \angle C = 90^0 \Rightarrow AC^2 + BC^2 = AB^2\ and$

$In\ \Delta CDE, \angle DCE = 90^0 \Rightarrow CD^2 + CE^2 = DE^2]$

$$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$$

Q14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see Fig. 6.55). Prove that $2AB^2 = 2AC^2 + BC^2$.

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

In ΔABC , $AD \perp BC$ and $BD = 3CD$

$$BD + CD = BC$$

$$3CD + CD = BC$$

$$4CD = BC$$

$$CD = \frac{1}{4}BC \text{………..(1)}$$

$$\text{and, } BD = \frac{3}{4}BC \text{………..(2)}$$

In ΔADC

$$AC^2 = AD^2 + CD^2 \qquad [\because \angle ADC = 90^0]$$

$$AD^2 = AC^2 - CD^2 \qquad \ldots\ldots\ldots(3)$$

In ΔADB

$$AB^2 = AD^2 + BD^2 \qquad [\because \angle ADB = 90^0]$$

$$AB^2 = AC^2 - CD^2 + BD^2 \qquad [\text{from (3)}]$$

$$AB^2 = AC^2 + \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 \qquad [\text{from (1) and (2)}]$$

$$AB^2 = AC^2 + \frac{9BC^2 - BC^2}{16}$$

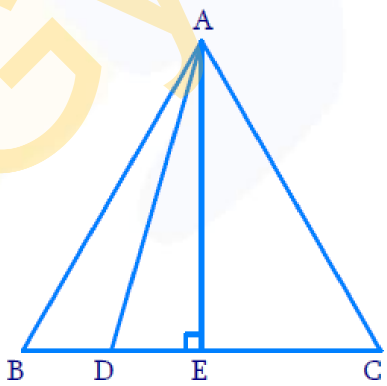
$$AB^2 = AC^2 + \frac{8BC^2}{16}$$

$$AB^2 = AC^2 + \frac{1}{2}BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Q15. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$ Prove that $9AD^2 = 7AB^2$.

Diagram



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

In ΔABC; $AB = BC = CA$ and $BD = \frac{1}{3}BC$

Draw $AE \perp BC$

$$BE = CE = \frac{1}{2}BC$$

[$\because$ In an equilateral triangle perpendicular drawn from vertex to opposite side bisects the side]

Now In ΔADE

$$AD^2 = AE^2 + DE^2 \text{ (Pythagoras theorem)}$$

$$= (\frac{\sqrt{3}}{2}BC)^2 + (BE - BD)^2$$

[$\because$ AE is the height of an equilateral triangle which is equal to $\frac{\sqrt{3}}{2}$ side]

$$AD^2 = \frac{3}{4}BC^2 + \left[\frac{BC}{2} - \frac{BC}{3}\right]^2$$

$$AD^2 = \frac{3}{4}BC^2 + \left(\frac{BC}{6}\right)^2$$

$$AD^2 = \frac{3}{4}BC^2 + \frac{BC^2}{36}$$

$$AD^2 = \frac{27BC^2 + BC^2}{36}$$

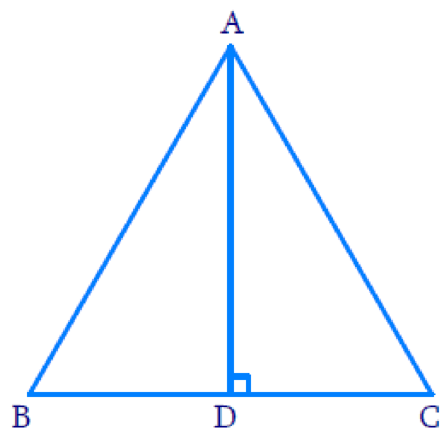
$$36AD^2 = 28BC^2$$

$$9AD^2 = 7BC^2$$

$$9AD^2 = 7AB^2 \ [\because AB = BC = CA]$$

Q16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Diagram



We have to prove $3BC^2 = 4AD^2$

Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution

In ΔABC

AB = BC = CA

$$AD \perp BC \Rightarrow BD = CD = \frac{BC}{2}$$

Now In ΔADC

$$AC^2 = AD^2 + CD^2$$

$$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 \left[AC = BC \text{ and } CD = \frac{BC}{2}\right]$$

$$BC^2 = AD^2 + \frac{BC^2}{4}$$

$$BC^2 - \frac{BC^2}{4} = AD^2$$

$$\frac{3BC^2}{4} = AD^2$$

$$3BC^2 - 4AD^2$$

Q17. Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm , AC = 12 cm and BC = 6 cm. The angle B is

(A) 120° (B) 60° (C) 90° (D) 45°

Reasoning:

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. Theorem 6.9

Solution:

(c)

In ΔABC

$$AB = 6\sqrt{3}\ cm; AC = 12\ cm; BC = 6\ cm$$

$$AB^2 = 108\ cm^2; AC^2 = 144\ cm^2; BC^2 = 36\ cm^2$$

$$AB^2 + BC^2 = (108 + 36)cm^2$$

$$= 144 cm^2$$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

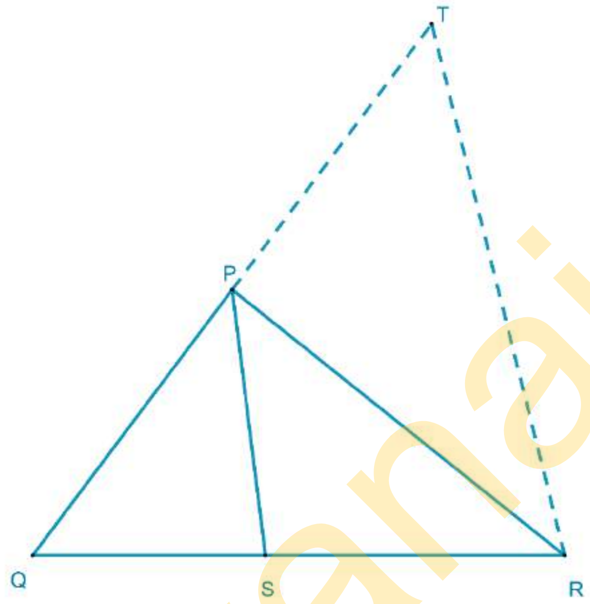
Pythagoras theorem satisfied

$$\Rightarrow \angle ABC = 90^0$$

Chapter - 6: Triangles

Exercise 6.6 (Page 152 of Grade 10 NCERT)

Q1. In Fig. 6.50, PS is the bisector of $\angle QPR$ of ΔPQR. Prove that $\frac{QS}{SR}=\frac{PQ}{PR}$



Reasoning:

As we know, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (BPT)

Solution:

Draw a line parallel to PS, through R, which intersect QP produced at T

PS|| RT

In ΔQPR

$\angle QPS=\angle SPR$ (Since PS is the bisector of $\angle QPR$)...............(i)

But $\angle PRT=\angle SPR$ (alternate interior angles)......................(ii)

$\angle QPS=\angle PTR$ (Corresponding angles)...............................(iii)

From (i), (ii), and (iii)

$\angle PTR=\angle PRT$

PR = PT .... (iv)

(Since in a triangle sides opposite to equal angles are equal)

In ΔQRT, $PS \parallel RT$

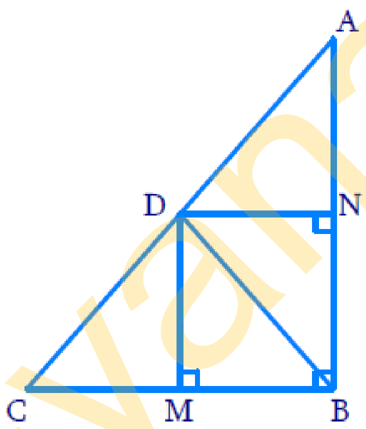
$$\frac{QS}{SR} = \frac{QP}{PT} \qquad [\text{BPT}]$$
$$\frac{QS}{SR} = \frac{QP}{PR} \qquad [\text{from (iv)}]$$

Q2. In Fig. 6.57, D is a point on hypotenuse AC of ΔABC, such that BD ⊥ AC, DM ⊥ BC and DN ⊥ AB.

Prove that:

(a) $DM^2 = DN.MC$

(b) $DN^2 = DM.AN$



Reasoning:
AA similarity criterion, BPT.

Solution:
(i) In quadrilateral $DMBN$
(ii)
$DM \perp BC$ and $DN \perp AB$

DMBN is a rectangle.

DM = BN and DN = BM ……...............(i)

In ΔDCM

$$\angle DCM + \angle DMC + \angle CDM = 180^\circ$$
$$\angle DCM + 90^\circ + \angle CDM = 180$$
$$\angle DCM + \angle CDM = 90^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

But $\angle CDM + \angle BDM = 90^\circ \ldots\ldots\ldots\ldots\ldots\ldots(iii)$

Since $BD \perp AC$

From (ii) and (iii)

$$\angle DCM = \angle BDM \ldots\ldots\ldots\ldots\ldots(\text{iv})$$

In ΔBDM

$$\angle DBM + \angle BDM = 90^{\circ} \ldots\ldots\ldots\ldots\ldots(\text{v})$$

Since, $DM \perp BC$

From (iii) and (v)

$$\angle CDM = \angle DBM \ldots\ldots\ldots\ldots(\text{vi})$$

Now in ΔDCM and ΔDBM

$\Delta DCM \sim \Delta BDM$ (From (iv) and (vi), AA criterion)

$\frac{DM}{BM} = \frac{MC}{DM}$ (Corresponding sides are in same ratio)

$DM^2 = BM.MC$

$DM^2 = DN.MC$ $\left[\text{from (i) } DN = BM\right]$

(iii) In ΔBDN

$\angle BDN + \angle DBN = 90^{\circ}$ (Since $DN \perp AB$)..................(vii)

But $\angle ADN + \angle BDN = 90^{\circ}$ (Since $BD \perp AC$)..................(viii)

From (vii) and (viii)

$\angle DBN = \angle ADN$..................(ix)

In ΔADN

$\angle DAN + \angle ADN = 90^{\circ}$ $(Since\ DN \perp AB)$..........(x)

But $\angle BDN + \angle ADN = 90^{\circ}$ (Since $BD \perp AC$)..........(xi)

From (xi) and (x)

$\angle DAN = \angle BDN$...................(xii)

Now in ΔBDN and ΔDAN ,

$\Delta BDN \sim \Delta DAN$ (From (ix) and (xii), AA criterion)

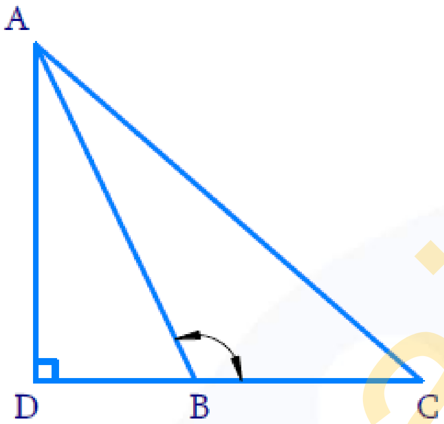
$$\frac{BN}{DN} = \frac{DN}{AN} \quad \text{(Corresponding sides are in same ratio)}$$

$$DN^2 = BN.AN$$

$$DN^2 = DM.AN \quad [\text{from (i) } BN = DM]$$

Q3. In Fig. 6.58, ABC is a triangle in which ∠ABC > 90° and AD ⊥ CB produced. Prove that:

$$AC^2 = AB^2 + BC^2 + 2BC.BD$$



Reasoning:

Pythagoras theorem

Solution:

In ΔADC

$$\angle ADC = 90^\circ$$

$$\Rightarrow AC^2 = AD^2 + CD^2$$

$$= AD^2 + [BD + BC]^2$$

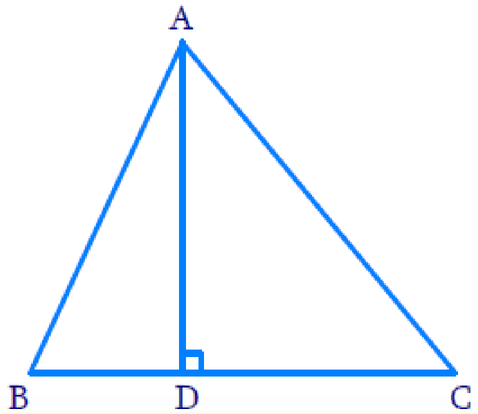
$$= AD^2 + BD^2 + BC^2 + 2BC \cdot BD$$

$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD \quad [\because \text{In } \Delta ADB,\ AB^2 = AD^2 + BD^2]$$

Q4. In Fig. 6.59, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC.

Prove that:

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$



Reasoning:

Pythagoras Theorem

Solution:

In ΔADC

$$\angle ADC = 90^\circ$$

$$\begin{aligned} AC^2 &= AD^2 + DC^2 \\ &= AD^2 + [BC - BD]^2 \\ &= AD^2 + BD^2 + BC^2 - 2BC.BD \\ AC^2 &= AB^2 + BC^2 - 2BC.BD \qquad [\because \text{In } \Delta ADB,\ AB^2 = AD^2 + BD^2] \end{aligned}$$

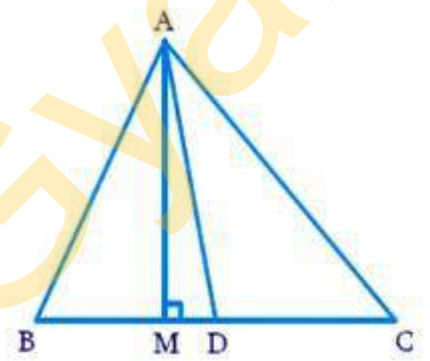
Q5. In Fig. 6.60, AD is a median of a triangle ABC and AM ⊥ BC.

Prove that:

i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

(i) In ΔAMC

$$\angle AMC = 90^\circ$$

$$\begin{aligned} AC^2 &= AM^2 + CM^2 \\ &= AM^2 + [DM + CD]^2 \\ &= AM^2 + DM^2 + CD^2 + 2DM.CD \\ &= AD^2 + \left(\frac{BC}{2}\right)^2 + 2DM\left(\frac{BC}{2}\right) \end{aligned}$$

Since, in ΔAMD, $AD^2 = AM^2 + DM^2$ and D is the midpoint of BC

means $BD = CD = \dfrac{BC}{2}$

$$AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 \text{..............(i)}$$

(ii) In $\Delta\, AMB$

$$\angle AMB = 90^\circ$$

$$AB^2 = AM^2 + BM^2$$
$$= AM^2 + [BD - DM]^2$$
$$= AM^2 + BD^2 + DM^2 - 2BD.DM$$
$$= AM^2 + DM^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)DM$$

Since, in ΔAMD, $AD^2 = AM^2 + DM^2$ and D is the midpoint of BC means $BD = CD = \dfrac{BC}{2}$

$$AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 \text{...............(ii)}$$

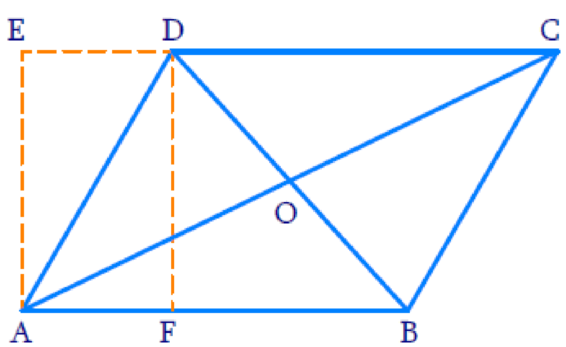
(iii) Adding (i) and (ii)

$$AC^2 + AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM + AD^2 + \left(\frac{BC}{2}\right)^2 - BC.DM$$

$$AC^2 + AB^2 = 2AD^2 + 2\left(\frac{BC}{2}\right)^2$$

$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$

Q6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.



Reasoning:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Theorem 6.8

Solution:

In parallelogram ABCD

AB = CD

AD = BC

Draw $AE \perp CD, \; DF \perp AB$

EA = DF (Perpendiculars drawn between same parallel lines)

In ΔAEC

$$AC^2 = AE^2 + EC^2$$
$$= AE^2 + [ED + DC]^2$$
$$= AE^2 + DE^2 + DC^2 + 2DE.DC$$
$$AC^2 = AD^2 + DC^2 + 2DE \cdot DC \ldots\ldots\ldots\ldots(i)$$
$$\left[\text{Since, } AD^2 = AE^2 + DE^2\right]$$

In ΔDFB

$$BD^2 = DF^2 + BF^2$$
$$= DF^2 + [AB - AF]^2$$
$$= DF^2 + AB^2 + AF^2 - 2AB.AF$$
$$= AD^2 + AB^2 - 2AB.AF$$
$$BD^2 = AD^2 + AB^2 - 2AB.AF \ldots\ldots\ldots\ldots\ldots..(ii)$$
$$\left[\text{Since, } AD^2 = DF^2 + AF^2\right]$$

Adding (i) and (ii)

$$AC^2 + BD^2 = AD^2 + DC^2 + 2DE.DC + AD^2 + AB^2 - 2AB.AF$$
$$AC^2 + BD^2 = BC^2 + DC^2 + 2AB.AF + AD^2 + AB^2 - 2AB.AF$$

(Since AD = BC and DE = AF, CD = AB)

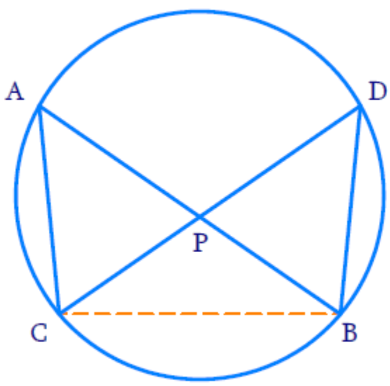
$$\Rightarrow AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$$

Q7. In Fig. 6.61, two chords AB and CD intersect each other at the point P.

Prove that:

(i) $\Delta APC \sim \Delta DPB$

(ii) AP. PB = CP. DP



Reasoning:

As we know that, two triangles, are similar if
(i) Their corresponding angles are equal and
(ii) Their corresponding sides are in the same ratio

As we know that angles in the same segment of a circle are equal.

Solution:

Draw BC

(i) In ΔAPC and ΔDPB

$\angle APC = \angle DPB$ (Vertically opposite angles)

$\angle PAC = \angle PDB$ (Angles in the same segment)

$\Rightarrow \Delta APC \sim \Delta DPB$ (A.A criterion)

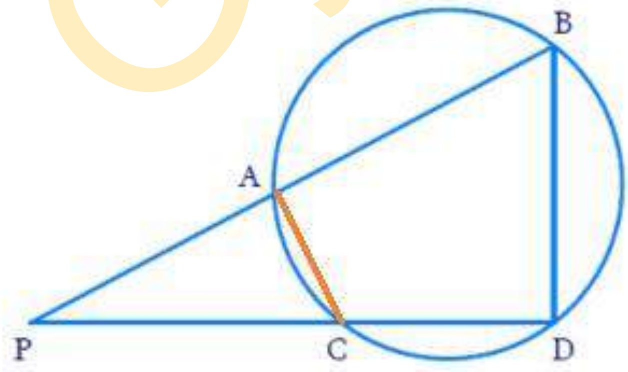
(ii) In ΔAPC and ΔDPB,

$$\frac{AP}{DP} = \frac{CP}{PB} = \frac{AC}{DB} \quad [\because \Delta APC \sim \Delta DPB]$$

$$\frac{AP}{DP} = \frac{CP}{PB}$$

$$\Rightarrow AP.PB = CP.DP$$

Q8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that
(i) $\Delta\, PAC \sim \Delta\, PDB$
(ii) PA. PB = PC. PD



Reasoning:

(i) Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
(ii) Basic proportionality theorem.

Solution:

Draw AC

(i) In ΔPAC and ΔPDB

$\angle APC = \angle BPD$ (Common angle)

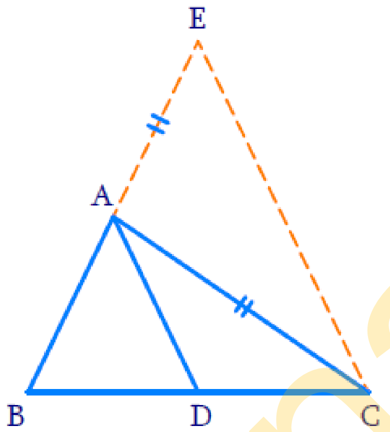
$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle)

$\Rightarrow \Delta PAC \sim \Delta PDB$

(ii) In ΔPAC and ΔPDB

$$\frac{PA}{PD}=\frac{PC}{PB}=\frac{AC}{BD}$$
$$\frac{PA}{PD}=\frac{PC}{PB}$$
$$PA.PB=PC.PD$$

Q9. In Fig. 6.63, D is a point on side BC of Δ ABC such that $\frac{BD}{CD}=\frac{BA}{CA}$ Prove that AD is the bisector of $\angle BAC$.



Reasoning:

(i) As we know that in an isosceles triangle, the angles opposite to equal sides are equal.

(ii) Converse of BPT.

Solution:

Extended BA to E such that AE = AC and join CE.

In ΔAEC

AE = AC $\Rightarrow \angle ACE=\angle AEC$ _____ (i)

It is given that

$$\frac{BD}{CD}=\frac{BA}{CA}$$
$$\frac{BD}{CD}=\frac{BA}{AE} \quad (\because AC=AE) \text{\_\_\_\_\_\_ (ii)}$$

In ΔABD and ΔEBC

$AD \parallel$ EC (Converse of BPT)

$\Rightarrow \angle BAD=\angle BEC$ (Corresponding angles) ________ (iii)

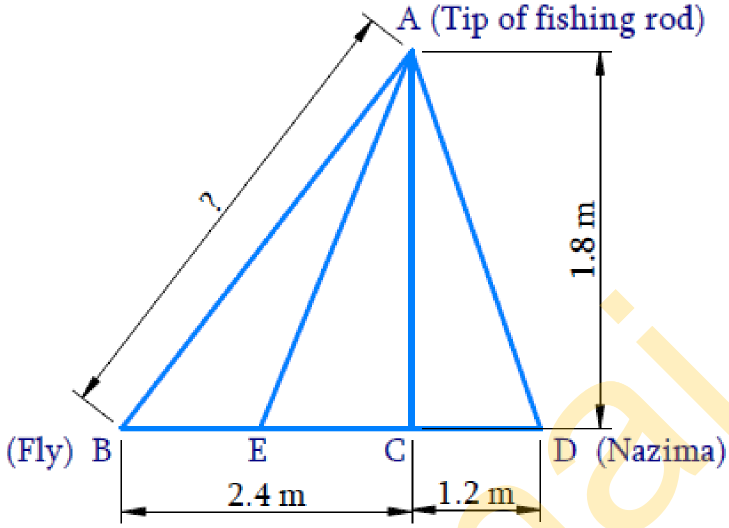
and $\angle DAC=\angle ACE$ (Alternate interior angles) ________ (iv)

From (i), (iii) and (iv)

$$\angle BAD=\angle DAC$$

$\Rightarrow AD$ is the bisector of $\angle BAC$

Q10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Reasoning:
Pythagoras Theorem

Solution:
To find AB and ED
BD = 3.6 m, BC = 2.4 m, CD = 1.2 m

AC = 1.8 cm

In ΔACB

$$AB^2 = AC^2 + BC^2$$
$$= (1.8)^2 + (2.4)^2$$
$$= 3.24 + 5.76$$
$$AB^2 = 9$$
$$AB = 3$$

Length of the string out AB= 3cm
Let the fly at E after 12 seconds
String pulled in 12 seconds = 12 × 5
= 60 cm
=0.6 m
AE = 3m – 0.6 m
= 2.4 m

Now In ΔACE

$$CE^2 = AE^2 - AC^2$$

$$= (2.4)^2 - (1.8)^2$$

$$CE^2 = 5.76 - 3.24$$

$$= 2.52$$

$$CE = 1.587m$$

DE = CE + CD

= 1.587 + 1.2

= 2.787

DE = 2.79 m

Horizontal distance of the fly after 12 seconds = 2.79 m