

## Chapter 8: Introduction to Trigonometry

### Exercise 8.1 (Page 181 of Grade 10 NCERT Textbook)

**Q1.** In  $\triangle ABC$ , right-angled at B, AB = 24 cm, BC = 7 cm, determine:

- (i)  $\sin A, \cos A$
- (ii)  $\sin C, \cos C$

**Difficulty level:** Easy

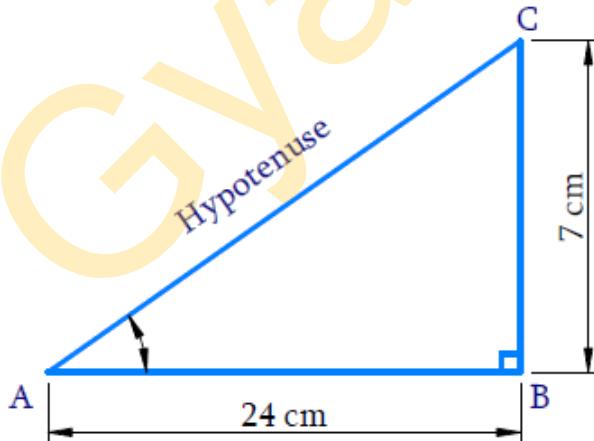
**What is the known?**

Two sides of a right-angled triangle  $\triangle ABC$

**What is the unknown?**

Sine and cosine of angle A and C.

**Reasoning:**



Applying Pythagoras theorem for  $\triangle ABC$ , we can find hypotenuse (side AC). Once hypotenuse is known, we can find sine and cosine angle using trigonometric ratios.

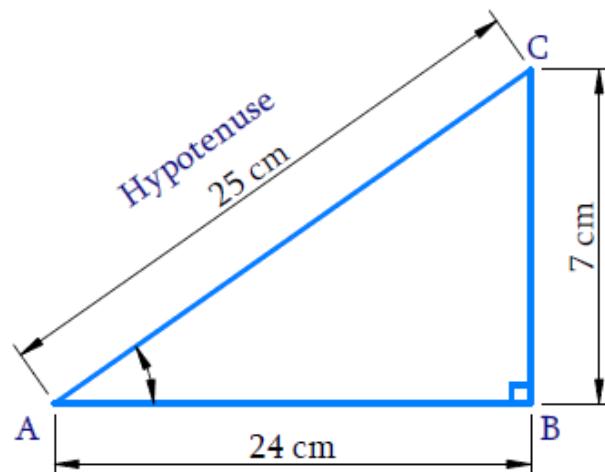
**Solution:**

In  $\triangle ABC$ , we obtain.

$$\begin{aligned}AC^2 &= AB^2 + BC^2 \\&= (24\text{cm})^2 + (7\text{cm})^2 \\&= (576 + 49)\text{ cm}^2 \\&= 625\text{ cm}^2\end{aligned}$$

$\therefore$  Hypotenuse  $AC = \sqrt{625} \text{ cm} = 25 \text{ cm}$

(i)



$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\sin A = \frac{7 \text{ cm}}{25 \text{ cm}} = \frac{7}{25}$$

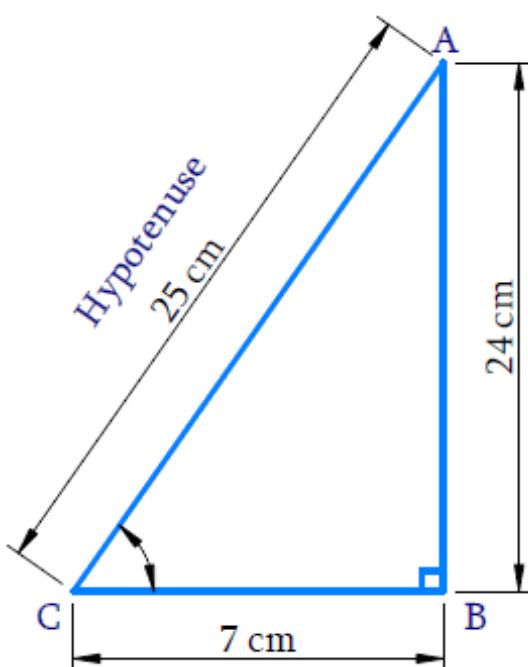
$$\sin A = \frac{7}{25}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24 \text{ cm}}{25 \text{ cm}} = \frac{24}{25}$$

$$\cos A = \frac{24}{25}$$

(ii)



$$\sin C = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\sin C = \frac{24 \text{ cm}}{25 \text{ cm}} = \frac{24}{25}$$

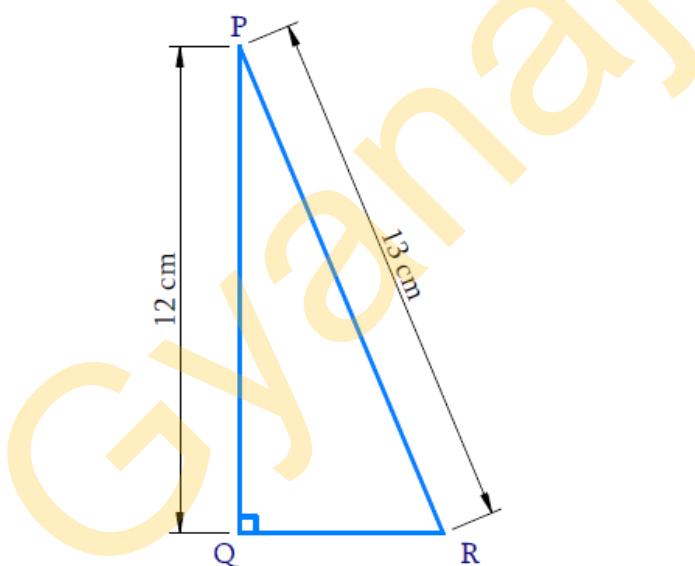
$$\sin C = \frac{24}{25}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7 \text{ cm}}{25 \text{ cm}} = \frac{7}{25}$$

$$\cos C = \frac{7}{25}$$

**Q2.** In the given figure, find  $\tan P - \cot R$ .



**Difficulty level:** Medium

**What is the known/given?**

$PQ = 12 \text{ cm}$  and  $PR = 13 \text{ cm}$ .

**What is the unknown?**

One side of right-angled triangle  $\Delta PQR$

**Reasoning:**

Using Pythagoras theorem, we can find the length of the third side. Then the required trigonometric ratios.

**Solution:**

Apply Pythagoras theorem for  $\Delta PQR$  we obtain:

$$PR^2 = PQ^2 + QR^2$$

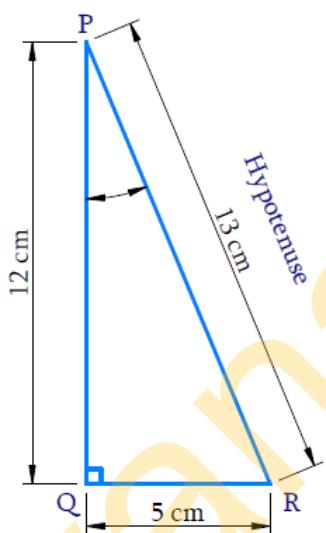
$$QR^2 = PR^2 - PQ^2$$

$$QR^2 = (13\text{cm})^2 - (12\text{cm})^2$$

$$QR^2 = 169\text{cm}^2 - 144\text{cm}^2$$

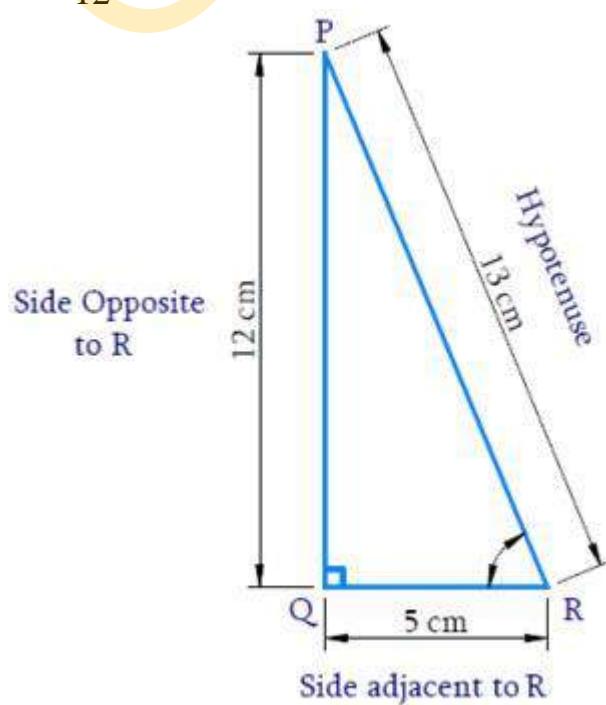
$$QR^2 = 25\text{cm}^2$$

$$QR = 5\text{cm}$$



$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{5\text{cm}}{12\text{cm}}$$

$$\tan P = \frac{5}{12}$$



$$\cot R = \frac{\text{side adjacent to } \angle R}{\text{side opposite to } \angle R} = \frac{QR}{PQ} = \frac{5\text{cm}}{12\text{cm}}$$

$$\cot R = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$

$$\tan P - \cot R = 0$$

**Q3.** If  $\sin A = \frac{3}{4}$  calculate  $\cos A$  and  $\tan A$ .

**Difficulty level:** Medium

**What is the known/given?**

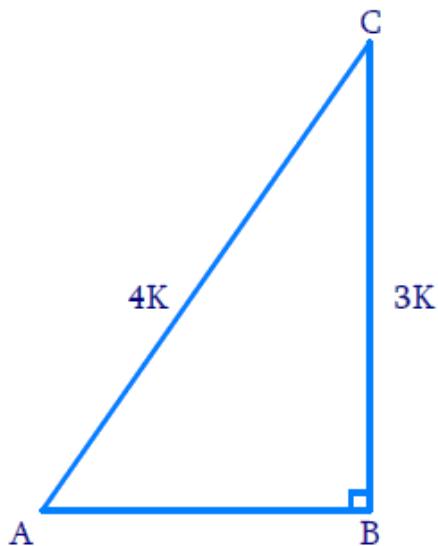
Sine of  $\angle A$ .

**What is the unknown?**

Cosine and tangent of  $\angle A$

**Reasoning:**

Using  $\sin A$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.



**Solution:**

Let  $\Delta ABC$  be a right-angled triangle, right angled at point B.

Given that

$$\sin A = \frac{3}{4}$$
$$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let BC be  $3k$ . Therefore, AC will be  $4k$  where  $k$  is a positive integer.

Applying Pythagoras theorem for  $\Delta ABC$ , we obtain:

$$AC^2 = AB^2 + BC^2$$
$$AB^2 = AC^2 - BC^2$$
$$AB^2 = (4k)^2 - (3k)^2$$
$$AB^2 = 16k^2 - 9k^2$$
$$AB^2 = 7k^2$$
$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k}$$
$$= \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k}$$
$$= \frac{3}{\sqrt{7}}$$

Thus,  $\cos A = \frac{\sqrt{7}}{4}$  and  $\tan A = \frac{3}{\sqrt{7}}$

**Q4.** Given  $15 \cot A = 8$ , find  $\sin A$  and  $\sec A$ .

**Difficulty level: Medium**

**What is the known/given?**

Cotangent of  $\angle A$

**What is the unknown?**

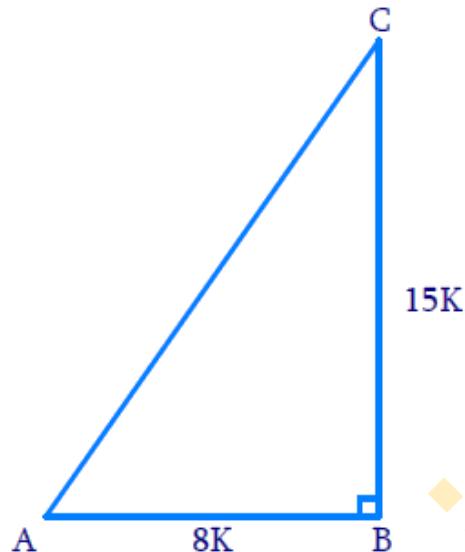
Sine and Secant of  $\angle A$ .

**Reasoning:**

Using  $\cot A$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

**Solution:**

Let us consider a right-angled  $\Delta ABC$ , right angled at B.



$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC}$$

It is given that

$$\cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB be  $8k$ . Therefore, BC will be  $15k$  where  $k$  is a positive integer.

Apply Pythagoras theorem in  $\Delta ABC$ , we obtain.

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (8k)^2 + (15k)^2$$

$$AC^2 = 64k^2 + 225k^2$$

$$AC^2 = 289k^2$$

$$AC = 17k$$

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k}$$

$$= \frac{15}{17}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AC}{AB} = \frac{17k}{8k}$$

$$= \frac{17}{8}$$

Thus,  $\sin A = \frac{15}{17}$  and  $\sec A = \frac{17}{8}$

**Q5.** Given  $\sec \theta = \frac{13}{12}$ , calculate all other trigonometric ratios.

**Difficulty level:** Medium

**What is the known/given?**

Secant of  $\theta$

**What is the unknown?**

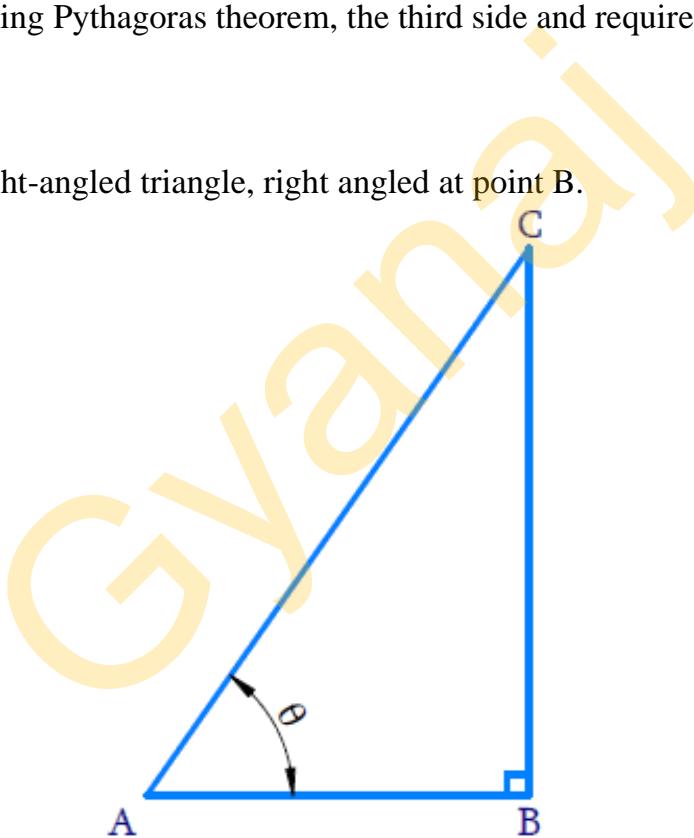
Other trigonometric ratios.

**Reasoning:**

Using  $\sec \theta$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

**Solution:**

Let  $\Delta ABC$  be a right-angled triangle, right angled at point B.



It is given that:

$$\sec \theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle \theta} = \frac{AC}{AB} = \frac{13}{12}$$

Let  $AC = 13k$  and  $AB = 12k$  where  $k$  is a positive integer.

Apply Pythagoras theorem in  $\Delta ABC$ , we obtain:

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = (13k)^2 - (12k)^2$$

$$BC^2 = 169k^2 - 144k^2$$

$$BC^2 = 25k^2$$

$$BC = 5k$$

$$\sin \theta = \frac{\text{side opposite to } \angle \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{side adjacent to } \angle \theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{side opposite to } \angle \theta}{\text{side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5}{12}$$

$$\cot \theta = \frac{\text{side adjacent to } \angle \theta}{\text{side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12}{5}$$

$$\cosec \theta = \frac{\text{hypotenuse}}{\text{side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13}{5}$$

**Q6.** If  $\angle A$  and  $\angle B$  are acute angles such that  $\cos A = \cos B$ , then show that  $\angle A = \angle B$ .

**Difficulty level:** Medium

**What is the known/given?**

$\angle A$  and  $\angle B$  are acute angles and  $\cos A = \cos B$ .

**What is the unknown?**

To show that  $\angle A = \angle B$

**Reasoning:**

Using  $\cos A$  and  $\cos B$ , we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then compare both the ratios.

### Solution:

In the right-angled triangle ABC,  $\angle A$  and  $\angle B$  are acute angles and  $\angle C$  is right angle.

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\cos B = \frac{\text{side adjacent to } \angle B}{\text{hypotenuse}} = \frac{BC}{AB}$$

Given that  $\cos A = \cos B$

Therefore,

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$AC = BC$$

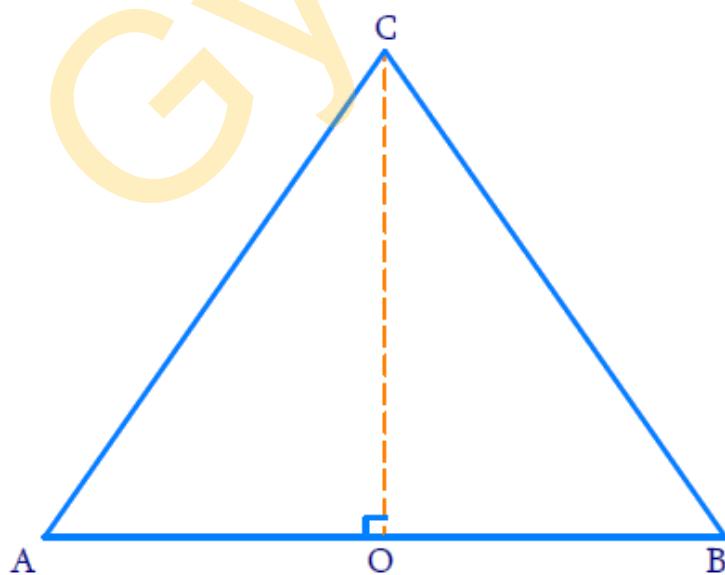
Hence,  $\angle A = \angle B$ , (angles opposite to equal sides of triangle are equal.)

**Alternatively,**

**Reasoning:**

Using  $\cos A$  and  $\cos B$ , we can find the ratio of the length of two sides of the right-angled triangle with respective angles. Then by using Pythagoras theorem, relation between the sides.

Let us consider a triangle ABC in which  $CO \perp AB$ .



It is given that

$$\cos A = \cos B$$

$$\frac{AO}{AC} = \frac{BO}{BC}$$

$$\frac{AO}{BO} = \frac{AC}{BC}$$

$$\text{Let } \frac{AO}{BO} = \frac{AC}{BC} = k$$

$$AO = k \cdot BO \quad (\text{i})$$

$$AC = k \cdot BC \quad (\text{ii})$$

By applying Pythagoras theorem in  $\Delta CAO$  and  $\Delta CBO$ , we get.

$$AC^2 = AO^2 + CO^2 \quad \text{from } \Delta CAO$$

$$CO^2 = AC^2 - AO^2 \quad (\text{iii})$$

$$BC^2 = BD^2 + CO^2 \quad \text{from } \Delta CBO$$

$$CO^2 = BC^2 - BO^2 \quad (\text{iv})$$

From equation (iii) and equation (iv), we get

$$AC^2 - AO^2 = BC^2 - BO^2$$

$$(kBC)^2 - (kBO)^2 = BC^2 - BO^2$$

$$k^2 BC^2 - k^2 BO^2 = BC^2 - BO^2$$

$$k^2 (BC^2 - BO^2) = BC^2 - BO^2$$

$$k^2 = \frac{BC^2 - BO^2}{BC^2 - BO^2} = 1$$

$$k = 1$$

Putting this value in equation (ii) we obtain

$$AC = BC$$

$\angle A = \angle B$  (angles opposite to equal sides of triangle are equal.)

**Q7.** If  $\cot \theta = \frac{7}{8}$ , evaluate: (i)  $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ , (ii)  $\cot^2 \theta$

**Difficulty Level:** Medium

**What is the known/given?**

$$\cot \theta = \frac{7}{8}$$

**What is the unknown?**

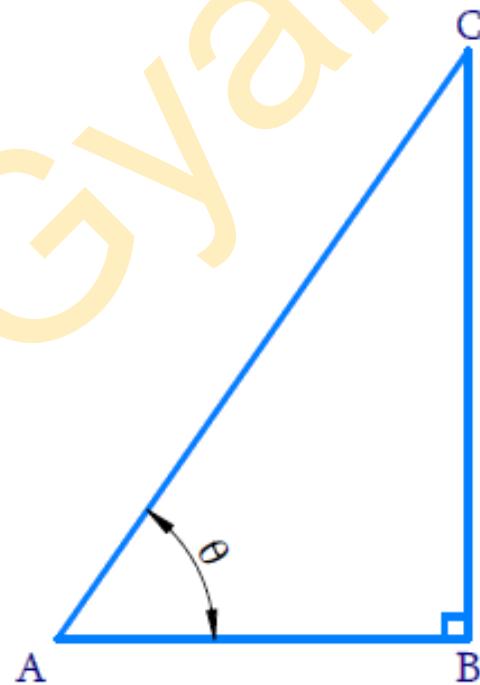
Value of (i)  $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ , and (ii)  $\cot^2 \theta$

**Reasoning:**

$\cot \theta = \frac{7}{8}$   
Using  $\cot \theta = \frac{7}{8}$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

**Solution:**

Let  $\Delta ABC$ , in which angle B is right angle.



$$\cot \theta = \frac{\text{side adjacent to } \theta}{\text{side opposite to } \theta} = \frac{AB}{BC} = \frac{7}{8}$$

Let  $AB = 7k$  and  $BC = 8k$ , where k is a positive integer.

By applying Pythagoras theorem in  $\Delta ABC$ , we get.

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 &= (7k)^2 + (8k)^2 \\
 &= 49k^2 + 64k^2 \\
 &= 113k^2
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{113k^2} \\
 &= \sqrt{113}k
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 \sin \theta &= \frac{\text{side opposite to } \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}} \\
 \cos \theta &= \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}
 \end{aligned}$$

$$\begin{aligned}
 (i) \quad & \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} = \frac{1-\sin^2 \theta}{1-\cos^2 \theta} \quad \left[ \because (a+b)(a-b) = (a^2 - b^2) \right] \\
 &= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} \\
 &= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} \\
 &= \frac{49/113}{64/113} \\
 &= \frac{49}{64}
 \end{aligned}$$

$$(ii) \quad \cot^2 \theta$$

$$\begin{aligned}
 \cot^2 \theta &= \left(\frac{7}{8}\right)^2 \\
 &= \frac{49}{64}
 \end{aligned}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

**Q8.** If  $3 \cot A = 4$ , check whether or not.

**Difficulty Level:** Medium

**What is the known/given?**

Cotangent of angle A

**What is the unknown?**

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

whether

**Reasoning:**

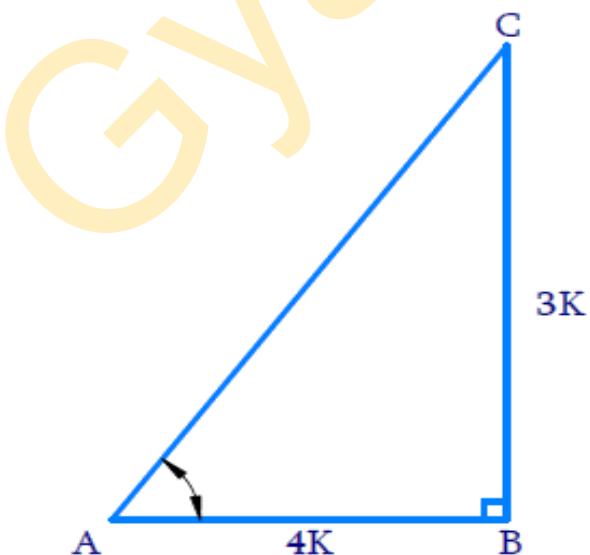
Using  $3\cot A = 4$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

**Solution:**

$$3\cot A = 4$$

$$\cot A = \frac{4}{3}$$

Let  $\Delta ABC$ , in which angle B is right angle.



$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC} = \frac{4}{3}$$

Let  $AB = 4k$  and  $BC = 3k$  where k is a positive integer.

By applying Pythagoras theorem in  $\Delta ABC$ , we get.

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 &= (4k)^2 + (3k)^2 \\
 &= 16k^2 + 9k^2 \\
 &= 25k^2
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{25k^2} \\
 &= 5k
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 \tan A &= \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4} \\
 \sin A &= \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5} \\
 \cos A &= \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 L.H.S &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \\
 &= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} \\
 &= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} \\
 &= \frac{16 - 9}{16 + 9} \\
 &= \frac{7}{25}
 \end{aligned}$$

$$\begin{aligned}
 R.H.S &= \cos^2 A - \sin^2 A \\
 &= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \\
 &= \frac{16}{25} - \frac{9}{25} \\
 &= \frac{16 - 9}{25} \\
 &= \frac{7}{25}
 \end{aligned}$$

$$\text{Therefore, } \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

- Q9.** In the triangle ABC right-angled at B, if  $\tan A = \frac{1}{\sqrt{3}}$  find the value of:
- (i)  $\sin A \cos C + \cos A \sin C$
  - (ii)  $\cos A \cos C - \sin A \sin C$

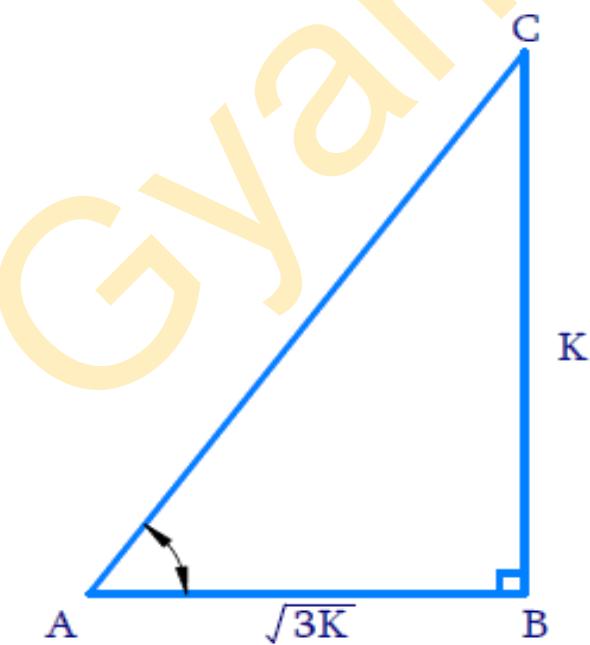
**Difficulty level: Medium**

**Reasoning:**

Using  $\tan A = \frac{1}{\sqrt{3}}$ , we can find the ratio of the length of two sides of the right-angled triangle. Then by using Pythagoras theorem, the third side and required trigonometric ratios.

**Solution:**

- (i) Let  $\Delta ABC$  be a right-angled triangle  $\tan A = \frac{1}{\sqrt{3}}$



$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let  $BC = k$  and  $AB = \sqrt{3}k$  where  $k$  is a positive real number.

By applying Pythagoras theorem for  $\Delta ABC$

$$\begin{aligned}
AC^2 &= AB^2 + BC^2 \\
&= (\sqrt{3}k)^2 + (k)^2 \\
&= 3k^2 + k^2 \\
&= 4k^2 \\
AC &= \sqrt{4k^2} \\
&= 2k
\end{aligned}$$

Therefore,

$$\begin{aligned}
\sin A &= \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2} \\
\cos A &= \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2} \\
\sin C &= \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2} \\
\cos C &= \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}
\end{aligned}$$

$$(i) \quad \sin A \cos C + \cos A \sin C$$

By substituting the values of the trigonometric functions in the above equation.

$$\begin{aligned}
\sin A \cos C + \cos A \sin C &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
&= \frac{1}{4} + \frac{3}{4} \\
&= \frac{1+3}{4} \\
&= \frac{4}{4} \\
&= 1
\end{aligned}$$

$$(ii) \quad \cos A \cos C - \sin A \sin C$$

By substituting the values of the trigonometric functions in the above equation.

$$\begin{aligned}
\cos A \cos C - \sin A \sin C &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
&= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\
&= 0
\end{aligned}$$

**Q10.** In  $\Delta PQR$ , right-angled at Q,  $PR + QR = 25 \text{ cm}$  and  $PQ = 5 \text{ cm}$ . Determine the values of  $\sin P$ ,  $\cos P$  and  $\tan P$ .

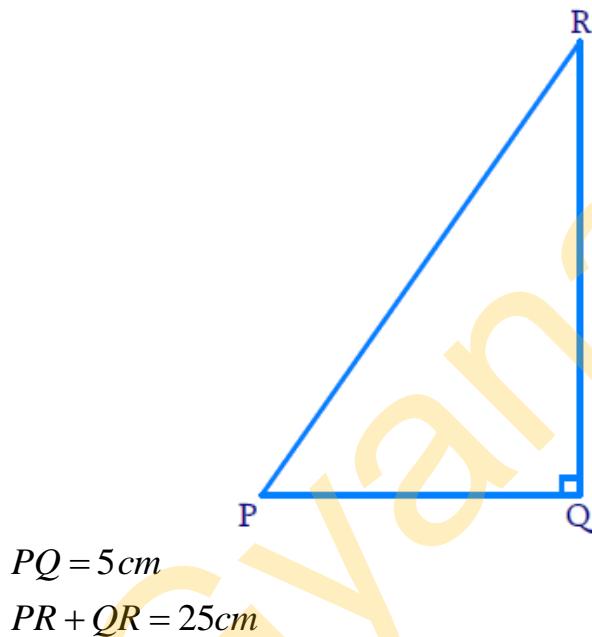
**Difficulty level:** Medium

**Reasoning:**

Using Pythagoras theorem, we can find the length of the all three sides. Then the required trigonometric ratios

**Solution:**

Given,  $\Delta PQR$  is right-angled at Q.



Let  $PR = x \text{ cm}$

Therefore,

$$\begin{aligned} QR &= 25\text{cm} - PR \\ &= (25 - x)\text{cm} \end{aligned}$$

By applying Pythagoras theorem for  $\Delta PQR$ , we obtain.

$$\begin{aligned} PR^2 &= PQ^2 + QR^2 \\ x^2 &= (5)^2 + (25 - x)^2 \\ x^2 &= 25 + 625 - 50x + x^2 \\ 50x &= 650 \\ x &= \frac{650}{50} \\ &= 13 \end{aligned}$$

Therefore,

$$PR = 13 \text{ cm}$$

$$\begin{aligned}QR &= (25 - 13) \text{ cm} \\&= 12 \text{ cm}\end{aligned}$$

By substituting the values obtained above in the trigonometric functions below.

$$\sin P = \frac{\text{side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

**Q11.** State whether the following are true or false. Justify your answer.

(i) The value of  $\tan A$  is always less than 1.

(ii)  $\sec A = \frac{12}{5}$  for some value of angle A.

(iii)  $\cos A$  is the abbreviation used for the cosecant of angle A.

(iv)  $\cot A$  is the product of cot and A.

(v)  $\sin \theta = \frac{4}{3}$ , for some angle  $\theta$ .

**Difficulty level: Medium**

**Solution:**

(i) False, because sides of a right-angled triangle may have any length. So  $\tan A$  may have any value.

(ii)

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A}$$

As hypotenuse is the largest side, the ratio on RHS will be greater than 1. Hence  $\sec A > 1$ . Thus, the given statement is true.

(iii) Abbreviation used for cosecant of  $\angle A$  is cosec A and  $\cos A$  is the abbreviation used for cosine of  $\angle A$ . Hence the given statement is false.

(iv)  $\cot A$  is not the product of cot and A. It is the cotangent of  $\angle A$ . Hence, the given statement is false.

$$(v) \sin \theta = \frac{4}{3}$$

We know that in a right-angled triangle,

$$\sin \theta = \frac{\text{side adjacent to } \angle \theta}{\text{hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Also, the value of Sine should be less than 1. Therefore, such value of  $\sin \theta$  is not possible. Hence the given statement is false.

Gyanaj

## Chapter 8: Introduction to Trigonometry

### Exercise 8.2 (Page 187 of Grade 10 NCERT Textbook)

**Q1.** Evaluate the following:

- |   |  |
|---|--|
| (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$<br>(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$<br>(v) $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ}$ | (ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$<br>(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}$ |
|---|--|

**Difficulty level: Medium**

**Reasoning:**

We know that,

Exact Values of Trigonometric Functions				
Angle ( $\theta$ )		$\sin (\theta)$	$\cos (\theta)$	$\tan (\theta)$
Degrees	Radians			
$0^\circ$	0	0	1	0
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^\circ$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ$	$\frac{\pi}{2}$	1	0	Not Defined

**Solution:**

(i)

$$\begin{aligned}
 \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\
 &= \frac{3}{4} + \frac{1}{4} \\
 &= \frac{3+1}{4} \\
 &= \frac{4}{4} \\
 &= 1
 \end{aligned}$$

(ii)

$$\begin{aligned}2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ &= 2(\tan 45^\circ)^2 + (\cos 30^\circ)^2 - (\sin 60^\circ)^2 \\&= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\&= 2 + \frac{3}{4} - \frac{3}{4} \\&= 2\end{aligned}$$

(iii)

$$\begin{aligned}\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} &= \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{1}\right)} \\&= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} \\&= \frac{1 \times \sqrt{3}}{\sqrt{2} \times (2+2\sqrt{3})} \\&= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}\end{aligned}$$

Multiplying numerator and denominator by  $\sqrt{2}(\sqrt{3}-1)$ , we get

$$\begin{aligned}&= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \frac{\sqrt{2}(\sqrt{3}-1)}{\sqrt{2}(\sqrt{3}-1)} \\&= \frac{3\sqrt{2}-\sqrt{6}}{4(3-1)} \\&= \frac{3\sqrt{2}-\sqrt{6}}{8}\end{aligned}$$

(iv)

$$\begin{aligned}\frac{\sin 30^{\circ} + \tan 45^{\circ} - \operatorname{cosec} 60^{\circ}}{\sec 30^{\circ} + \cos 60^{\circ} + \cot 45^{\circ}} &= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} \\&= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} \\&= \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + 3\sqrt{3}}{2\sqrt{3}}} \\&= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}\end{aligned}$$

Multiplying numerator and denominator by  $(3\sqrt{3} - 4)$ , we get

$$\begin{aligned}&= \frac{(3\sqrt{3} - 4)(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)} \\&= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} \\&= \frac{43 - 24\sqrt{3}}{11}\end{aligned}$$

(v)

$$\begin{aligned}\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} &= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (-1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\&= \frac{\left(\frac{5}{4} + \frac{16}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)} \\&= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{3+1}{4}\right)} \\&= \frac{\left(\frac{67}{12}\right)}{\left(\frac{4}{4}\right)} \\&= \frac{67}{12}\end{aligned}$$

**Q2.** Choose the correct option and justify your choice:

(i)  $\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$

- (A)  $\sin 60^\circ$  (B)  $\cos 60^\circ$  (C)  $\tan 60^\circ$  (D)  $\sin 60^\circ$

(ii)  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

- (A)  $\tan 90^\circ$  (B) 1 (C)  $\sin 45^\circ$  (D)  $0^\circ$

(iii)  $\sin 2A = 2\sin A$  is true when  $A =$

- (A)  $0^\circ$  (B)  $30^\circ$  (C)  $45^\circ$  (D)  $60^\circ$

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

(A)  $\cos 60^\circ$     (B)  $\sin 60^\circ$     (C)  $\tan 60^\circ$     (D)  $\sin 30^\circ$

**Difficulty level:** Medium

**Reasoning:**

We know that,

Exact Values of Trigonometric Functions				
Angle ( $\theta$ )		$\sin (\theta)$	$\cos (\theta)$	$\tan (\theta)$
Degrees	Radians			
$0^\circ$	0	0	1	0
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^\circ$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ$	$\frac{\pi}{2}$	1	0	Not Defined

**Solution:**

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

By substituting the values of given trigonometric ratios in the above equation, we get.

$$= \frac{2 \times \left( \frac{1}{\sqrt{3}} \right)}{1 + \left( \frac{1}{\sqrt{3}} \right)^2}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

Out of the given options only  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ . Hence, option (A) is correct.

(ii)

$$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$$

By substituting the values of given trigonometric ratios for  $\tan 45^\circ$ .

$$\begin{aligned} &= \frac{1 - (1)^2}{1 + (1)^2} \\ &= \frac{1 - 1}{1 + 1} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$

Hence, option (D) is correct.

(iii)

$$\sin 2A = 2 \sin A$$

By substituting  $A = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$ , we get

For  $A = 0^\circ$

$$\begin{aligned} \sin 2A &= \sin 2 \times 0^\circ \\ &= \sin 0^\circ \\ &= 0 \\ 2 \sin A &= 2 \times \sin 0^\circ \\ &= 2 \times 0^\circ \\ &= 0 \end{aligned}$$

$$\sin 2A = 2 \sin A \quad (\text{When } A = 0^\circ)$$

For  $A = 30^\circ$

$$\sin 2A = \sin 2 \times 30^\circ$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$2 \sin A = 2 \times \sin 30^\circ$$

$$= 2 \times \frac{1}{2}$$

$$= 1$$

$$\sin 2A \neq 2 \sin A \quad (\text{When } A = 30^\circ)$$

For  $A = 45^\circ$

$$\begin{aligned}\sin 2A &= \sin 2 \times 45^\circ \\&= \sin 90^\circ \\&= 1 \\2 \sin A &= 2 \times \sin 45^\circ \\&= 2 \times \frac{1}{\sqrt{2}} \\&= \sqrt{2} \\2 \sin A &\neq 2 \sin A \quad (\text{When } A = 45^\circ)\end{aligned}$$

For  $A = 60^\circ$

$$\begin{aligned}\sin 2A &= \sin 2 \times 60^\circ \\&= \sin 120^\circ \\&= \frac{\sqrt{3}}{2} \\2 \sin A &= 2 \times \sin 60^\circ \\&= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} \\2 \sin A &\neq 2 \sin A \quad (\text{When } A = 60^\circ)\end{aligned}$$

Hence Option (A) is correct

(iv)

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

By substituting the values of given trigonometric ratios for  $\tan 30^\circ$ , we get

$$\begin{aligned}&= \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\&= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(1 - \frac{1}{3}\right)} \\&= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{3}\right)} \\&= \frac{2}{\sqrt{3}} \times \frac{3}{2} \\&= \sqrt{3}\end{aligned}$$

Out of the given option only  $\tan 60^\circ = \sqrt{3}$ .

Hence option (C) is correct.

**Q3.** If  $\tan(A+B) = \sqrt{3}$  and  $\tan(A-B) = \frac{1}{\sqrt{3}}$ ;  $0^\circ < (A+B) \leq 90^\circ$ ,  $A > B$ , find A and B.

**Difficulty level:** Medium

**Solution:**

Given that

$$\tan(A+B) = \sqrt{3} \text{ and, } \tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\text{Since, } \tan 60^\circ = \sqrt{3} \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Therefore,

$$\tan(A+B) = \tan 60^\circ$$

$$(A+B) = 60^\circ \quad (\text{i})$$

$$\tan(A-B) = \tan 30^\circ$$

$$(A-B) = 30^\circ \quad (\text{ii})$$

On adding both equations (i) and (ii), we obtain:

$$A+B+A-B=60^\circ+30^\circ$$

$$2A=90^\circ$$

$$A=45^\circ$$

By substituting the value of A in equation (i) we obtain

$$A+B=60^\circ$$

$$45^\circ+B=60^\circ$$

$$B=60^\circ-45^\circ=15^\circ$$

Therefore,  $\angle A = 45^\circ$  and  $\angle B = 15^\circ$  ( $A > B$ )

**Q4.** State whether the following are true or false. Justify your answer.

- (i)  $\sin(A+B) = \sin A + \sin B$ .
- (ii) The value of  $\sin \theta$  increases as  $\theta$  increases.
- (iii) The value of  $\cos \theta$  increases as  $\theta$  increases.
- (iv)  $\sin \theta = \cos \theta$  for all values of  $\theta$ .
- (v)  $\cot A$  is not defined for  $A = 0^\circ$ .

**Difficulty level:** Medium

**Solution:**

$$\sin(A + B) = \sin A + \sin B.$$

For the purpose of verification, Let  $A = 30^\circ$  and  $B = 60^\circ$

$$\begin{aligned} L.H.S &= \sin(A + B) \\ &= \sin(30^\circ + 60^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} R.H.S &= \sin A + \sin B \\ &= \sin 30^\circ + \sin 60^\circ \\ &= \frac{1}{2} + \frac{\sqrt{3}}{2} \\ &= \frac{1 + \sqrt{3}}{2} \end{aligned}$$

Since,  $\sin(A + B) \neq \sin A + \sin B$ .

Hence, the given statement is not true

(ii) The value of  $\sin \theta$  increases from 0 to 1 as  $\theta$  increases from  $0^\circ$  to  $90^\circ$

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

(iii) The value of  $\cos \theta$  decreases from 1 to 0 as  $\theta$  increases from  $0^\circ$  to  $90^\circ$

$$\cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

Hence, the given statement is false.

(iv)

This is true when  $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}} \text{ and } \cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for other values of  $\theta$

As,

$$\sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and } \cos 60^\circ = \frac{1}{2}$$

$$\sin 90^\circ = 1 \text{ and } \cos 90^\circ = 0$$

Hence, the given statement is false.

(v)

$$\cot A = \frac{\cos A}{\sin A}$$

$$\therefore \cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined}$$

Hence the given statement is true.

## Chapter 8: Introduction to Trigonometry

### Exercise 8.3 (Page 189 of Grade 10 NCERT Textbook)

**Q1.** Evaluate:

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ}$       (ii)  $\frac{\tan 26^\circ}{\cot 64^\circ}$       (iii)  $\cos 48^\circ - \sin 42^\circ$       (iv)  $\operatorname{cosec} 31^\circ - \sec 59^\circ$

**Difficulty level: Medium**

**Reasoning:**

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

**Solution:**

(i)

$$\frac{\sin 18^\circ}{\cos 72^\circ}$$

Since,

$$\sin(90^\circ - \theta) = \cos \theta$$

Here  $\theta = 72^\circ$

$$\begin{aligned}&= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} \\&= \frac{\cos 72^\circ}{\cos 72^\circ} \\&= 1\end{aligned}$$

(ii)

$$\frac{\tan 26^\circ}{\cot 64^\circ}$$

Since

$$\tan(90^\circ - \theta) = \cot \theta$$

Here  $\theta = 64^\circ$

$$\begin{aligned}
 &= \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ} \\
 &= \frac{\cot 64^\circ}{\cot 64^\circ} \\
 &= 1
 \end{aligned}$$

(iii)

$$\cos 48^\circ - \sin 42^\circ$$

Since,

$$\sin(90^\circ - \theta) = \cos \theta$$

Here  $\theta = 48^\circ$

$$\begin{aligned}
 &= \cos 48^\circ - \sin(90^\circ - 48^\circ) \\
 &= \cos 48^\circ - \cos 48^\circ \\
 &= 0
 \end{aligned}$$

(iv)

$$\operatorname{cosec} 31^\circ - \sec 59^\circ$$

Since,

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

Here  $\theta = 31^\circ$

$$\begin{aligned}
 &= \operatorname{cosec} 31^\circ - \sec(90^\circ - 31^\circ) \\
 &= \operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ \\
 &= 0
 \end{aligned}$$

**Q2.** Show that:

- (i)  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$
- (ii)  $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

**Difficulty level: Medium**

**Reasoning:**

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

**Solution:**

(i) Taking L.H.S

$$= \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

Since  $\tan(90^\circ - \theta) = \cot \theta$

$$= \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\cot 42^\circ \tan 42^\circ)(\cot 67^\circ \tan 67^\circ)$$

$$= \left( \frac{1}{\tan 42^\circ} \times \tan 42^\circ \right) \left( \frac{1}{\tan 67^\circ} \times \tan 67^\circ \right)$$

$$= 1 \times 1$$

$$= 1$$

$$= \text{R.H.S}$$

Hence,  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) Taking L.H.S

$$= \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

Since,  $\sin(90^\circ - \theta) = \cos \theta$

$$= \cos 38^\circ \cos 52^\circ - \sin(90^\circ - 52^\circ) \sin(90^\circ - 38^\circ)$$

$$= \cos 38^\circ \cos 52^\circ - \cos 52^\circ \cos 38^\circ$$

$$= 0$$

$$= \text{R.H.S}$$

Hence,  $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

**Q3.** If  $\tan 2A = \cot(A - 18^\circ)$ , where  $2A$  is an acute angle, find the value of  $A$ .

**Difficulty level: Medium**

**Reasoning:**

$$\tan(90^\circ - \theta) = \cot \theta$$

**Solution:**

Given that:  $\tan 2A = \cot(A - 18^\circ)$  ....(i)

But  $\tan 2A = \cot(90^\circ - 2A)$

By substituting this in equation (i) we get:

$$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$$

$$90^\circ - 2A = A - 18^\circ$$

$$3A = 108^\circ$$

$$A = \frac{108^\circ}{3} =$$

$$A = 36^\circ$$

**Q4.** If  $\tan A = \cot B$ , prove that  $A + B = 90^\circ$ .

**Difficulty level:** Easy

**Reasoning:**

$$\tan(90^\circ - \theta) = \cot \theta$$

**Solution:**

Given that:  $\tan A = \cot B$  (i)

We know that  $\tan A = \cot(90^\circ - A)$

By substituting this in equation (i) we get:

$$\cot(90^\circ - A) = \cot B$$

$$90^\circ - A = B$$

$$A + B = 90^\circ$$

**Q5.** If  $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ , where  $4A$  is an acute angle, find the value of  $A$ .

**Difficulty level:** Easy

**Reasoning:**

$$\sec A = \operatorname{cosec}(90^\circ - A)$$

**Solution:**

Given that:  $\sec 4A = \operatorname{cosec}(A - 20^\circ)$  ....(i)

Since,  $\sec A = \operatorname{cosec}(90^\circ - A)$

By using property in equation (i) we get:

$$\operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$$

$$90^\circ - 4A = A - 20^\circ$$

$$5A = 110^\circ$$

$$A = \frac{110^\circ}{5}$$

$$A = 22^\circ$$

**Q6.** If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

**Difficulty level: Medium**

**Reasoning:**

$$\sin(90^\circ - \theta) = \cos \theta$$

**Solution:**

We know that for  $\triangle ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

On dividing both sides by 2, we get:

$$\frac{\angle B + \angle C}{2} = \frac{180^\circ - \angle A}{2}$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

Applying sine angles on both the sides:

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

Since

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\therefore \sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$$

**Q7.** Express  $\sin 67^0 + \cos 75^0$  in terms of trigonometric ratios of angles between  $0^0$  and  $45^0$ .

**Difficulty level:** Medium

**Reasoning:**

$$\cos(90^0 - \theta) = \sin \theta$$

**Solution:**

Given that:  $\sin 67^0 + \cos 75^0 \dots \text{(i)}$

Since  $\cos(90^0 - \theta) = \sin \theta$

By using property in equation (i) we get:

$$\begin{aligned} &= \sin(90^0 - 23^0) + \cos(90^0 - 15^0) \\ &= \cos 23^0 + \sin 15^0 \end{aligned}$$

Hence, the expression  $\cos 23^0 + \sin 15^0$  has trigonometric ratios of angles between  $0^0$  and  $45^0$ .

## Chapter 8: Introduction to Trigonometry

### Exercise 8.4 (Page 193 of Grade 10 NCERT Textbook)

**Q1.** Express the trigonometric ratios  $\sin A$ ,  $\sec A$  and  $\tan A$  in terms of  $\cot A$ .

**Difficulty level:** Medium

**Reasoning:**

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sec^2 A = 1 + \tan^2 A$$

**Solution:**

Consider a  $\Delta ABC$  with  $\angle B = 90^\circ$

Using the Trigonometric Identity,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

(By taking reciprocal both the sides)

$$\left( \text{As } \frac{1}{\operatorname{cosec}^2 A} = \sin^2 A \right)$$

Therefore,

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

For any sine value with respect to an angle in a triangle, sine value will never be negative. Since, sine value will be negative for all angles greater than  $180^\circ$ .

$$\text{Therefore, } \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\text{We know that, } \tan A = \frac{\sin A}{\cos A}$$

$$\text{However, Trigonometric Function, } \cot A = \frac{\cos A}{\sin A}$$

$$\text{Therefore, Trigonometric Function, } \tan A = \frac{1}{\cot A}$$

$$\text{Also, } \sec^2 A = 1 + \tan^2 A \quad (\text{Trigonometric Identity})$$

$$\begin{aligned}
 &= 1 + \frac{1}{\cot^2 A} \\
 &= \frac{\cot^2 A + 1}{\cot^2 A} \\
 \sec A &= \frac{\sqrt{\cot^2 A + 1}}{\cot A}
 \end{aligned}$$

**Q2.** Write all the other trigonometric ratios of  $\angle A$  in terms of  $\sec A$ .

**Difficulty level:** Medium

**Reasoning:**

$$\begin{aligned}
 \sin^2 A + \cos^2 A &= 1 \\
 \operatorname{cosec}^2 A &= 1 + \cot^2 A \\
 \sec^2 A &= 1 + \tan^2 A
 \end{aligned}$$

**Solution:**

We know that,

$$\text{Trigonometric Function, } \cos A = \frac{1}{\sec A} \dots \text{Equation (1)}$$

Also,

$$\begin{aligned}
 \sin^2 A + \cos^2 A &= 1 \text{ (Trigonometric identity)} \\
 \sin^2 A &= 1 - \cos^2 A \text{ (By transposing)}
 \end{aligned}$$

Using value of  $\cos A$  from Equation (1) and simplifying further,

$$\begin{aligned}
 \sin A &= \sqrt{1 - \left( \frac{1}{\sec A} \right)^2} \\
 &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} \\
 &= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \dots \text{Equation (2)}
 \end{aligned}$$

$$\tan^2 A + 1 = \sec^2 A \text{ (Trigonometric identity)}$$

$$\tan^2 A = \sec^2 A - 1 \text{ (By transposing)}$$

Trigonometric Function,

$$\tan A = \sqrt{\sec^2 A - 1} \quad \dots \text{Equation (3)}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$= \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} \quad \dots (\text{By substituting Equations (1) and (2)})$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$= \frac{\sec A}{\sqrt{\sec^2 A - 1}} \quad (\text{By substituting Equation (2) and simplifying})$$

**Q3.** Evaluate

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

**Difficulty level:** Medium

**Reasoning:**

$$\sin^2 A + \cos^2 A = 1$$

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

**Solution:**

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} \quad (\sin(90^\circ - \theta) = \cos \theta \text{ & } \cos(90^\circ - \theta) = \sin \theta)$$

$$= \frac{1}{1} \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1)$$

$$= 1$$

$$\begin{aligned}
 & \text{(ii)} \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ \\
 &= \sin 25^\circ [\cos(90^\circ - 25^\circ)] + \cos 25^\circ [\sin(90^\circ - 25^\circ)] \\
 &= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ \quad [ \because \sin(90^\circ - \theta) = \cos \theta \text{ & } \cos(90^\circ - \theta) = \sin \theta ] \\
 &= \sin^2 25^\circ + \cos^2 25^\circ \\
 &= 1 \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1)
 \end{aligned}$$

**Q4.** Choose the correct option. Justify your choice.

(i)  $9 \sec 2A - 9 \tan 2A = \underline{\hspace{2cm}}$

- (A) 1
- (B) 9
- (C) 8
- (D) 0

(ii)  $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

- (A) 0
- (B) 1
- (C) 2
- (D) -1

(iii)  $(\sec A + \tan A)(1 - \sin A) = \underline{\hspace{2cm}}$

- (A)  $\sec A$
- (B)  $\sin A$
- (C)  $\operatorname{cosec} A$
- (D)  $\cos A$

(iv)  $\frac{1+\tan^2 A}{1+\cot^2 A}$

- (A)  $\sec 2A$
- (B) -1
- (C)  $\cot 2A$
- (D)  $\tan 2A$

**Difficulty level: Medium**

**Reasoning:**

$$\sin^2 A + \cos^2 A = 1$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sec^2 A = 1 + \tan^2 A$$

**Solution:**

$$(i) 9 \sec^2 A - 9 \tan^2 A$$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 \text{ [By the identity, } 1 + \sec^2 A = \tan^2 A, \text{ Hence } \sec^2 A - \tan^2 A = 1]$$

$$= 9$$

$$(ii) (1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$$

.....Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$\Rightarrow \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \quad (\text{By taking LCM and multiplying})$$

$$= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta} \quad (\text{Using } a^2 - b^2 = (a+b)(a-b))$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \quad (\text{Using identify } \sin^2 \theta + \cos^2 \theta = 1)$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

Hence, option(C) is correct.

$$(iii) (\sec A + \tan A) (1 - \sin A) \dots\dots(1)$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned} &= \left( \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A) \\ &= \left( \frac{1 + \sin A}{\cos A} \right) (1 - \sin A) \\ &= \frac{1 - \sin^2 A}{\cos A} \\ &= \frac{\cos^2 A}{\cos A} \quad (\text{By identify } \sin^2 \theta + \cos^2 \theta = 1, \text{ Hence } 1 - \sin^2 \theta = \cos^2 \theta) \\ &= \cos A \end{aligned}$$

Hence, option (D) is correct.

$$(iv) \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$= \frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned}
\frac{1+\tan^2 A}{1+\cot^2 A} &= \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}} \\
&= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} \\
&= \frac{1}{\frac{1}{\sin^2 A}} \\
&= \frac{\sin^2 A}{\cos^2 A} \\
&= \tan^2 A
\end{aligned}$$

Hence, option (D) is correct.

**Q5.** Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) (\cosec \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cosec \theta$$

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A + 1} = \cosec A + \cot A$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \cosec A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$(ix) (\cosec A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$(x) \left( \frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left( \frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

**Difficulty level: Medium**

**Reasoning:**

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \\ \sec^2 A &= 1 + \tan^2 A\end{aligned}$$

**Solution:**

(i)  $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$   
L.H.S =  $(\operatorname{cosec} \theta - \cot \theta)^2 \dots\dots\dots(1)$

We know that the trigonometric functions,

$$\begin{aligned}\cot(x) &= \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)} \\ \operatorname{cosec}(x) &= \frac{1}{\sin(x)}\end{aligned}$$

By substituting the above function in Equation (1)

$$\begin{aligned}(\operatorname{cosec} \theta - \cot \theta)^2 &= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\ &= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} \\ &= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\ &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1 \text{ Hence, } 1 - \cos^2 A = \sin^2 A) \\ &= \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} \quad [\text{Using } a^2 - b^2 = (a+b)(a-b)] \\ &= \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \text{RHS}\end{aligned}$$

$$(ii) \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$$

$$\begin{aligned}
 L.H.S &= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} \\
 &= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)} \\
 &= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{(1+\sin A)(\cos A)} \\
 &= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)} \\
 &= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} \quad (\text{By identify } \sin^2 A + \cos^2 A = 1) \\
 &= \frac{2+2\sin A}{(1+\sin A)(\cos A)} \\
 &= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} \\
 &= \frac{2}{\cos A} \\
 &= 2 \sec A \\
 &= R.H.S
 \end{aligned}$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cosec \theta$$

$$LHS = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \quad \dots\dots(1)$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

By substituting the above relations in Equation (1),

$$\begin{aligned}
&= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\cos \theta}{\sin \theta}} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
&= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\
&= \frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\
&= \frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right]
\end{aligned}$$

Using  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\begin{aligned}
&= \frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \right] \\
&= \frac{(1 + \sin \theta \cos \theta)}{(\sin \theta \cos \theta)} \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1) \\
&= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \\
&= 1 + \sec \theta \cosec \theta \\
&= \text{R.H.S.}
\end{aligned}$$

$$(iv) \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{L.H.S} = \frac{1 + \sec A}{\sec A} \quad \dots \dots (1)$$

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned}
 \frac{1+\sec A}{\sec A} &= \frac{\frac{1}{\cos A} + 1}{\frac{1}{\cos A}} \\
 &= \frac{\cos A + 1}{\cos A} \\
 &= \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{\cos A} \\
 &= (1 + \cos A)
 \end{aligned}$$

By multiplying  $(1 - \cos A)$ , in both denominator and numerator

$$\begin{aligned}
 &\Rightarrow \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)} \\
 &= \frac{1 - \cos^2 A}{1 - \cos A} \\
 &= \frac{\sin^2 A}{1 - \cos A} \quad [\text{By Identity } \sin^2 A + \cos^2 A = 1] \\
 &= R. H. S
 \end{aligned}$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$L.H.S = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Diving both numerator and denominator by  $\sin A$

$$\begin{aligned}
 &\Rightarrow \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}
 \end{aligned}$$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

We get

$$\begin{aligned} &\Rightarrow \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\ &\Rightarrow \frac{\cot A - (1 - \operatorname{cosec} A)}{\cot A + (1 - \operatorname{cosec} A)} \end{aligned}$$

We know that,  $1 + \cot^2 A = \operatorname{Cosec}^2 A$

Hence multiplying  $[\cot A - (1 - \operatorname{cosec} A)]$  in numerator and denominator

$$\begin{aligned} &\Rightarrow \frac{[(\cot A) - (1 - \operatorname{cosec} A)][(\cot A) - (1 - \operatorname{cosec} A)]}{[(\cot A) + (1 - \operatorname{cosec} A)][(\cot A) - (1 - \operatorname{cosec} A)]} \\ &= \frac{[\cot A - (1 - \operatorname{cosec} A)]^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \\ &= \frac{\cot^2 A + (1 - \operatorname{cosec} A)^2 - 2\cot A(1 - \operatorname{cosec} A)}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)} \\ &= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)} \\ &= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A} \\ &= \frac{2\operatorname{cosec} A (\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)} \\ &= \operatorname{cosec} A + \cot A \\ &= \text{R.H.S} \end{aligned}$$

$$(vi) \sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$$LHS = \sqrt{\frac{1+\sin A}{1-\sin A}} \quad \dots\dots (1)$$

Multiplying and dividing by  $\sqrt{(1+\sin A)}$

$$\begin{aligned} &\Rightarrow \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} \\ &= \sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}} \quad [ \because a^2 - b^2 = (a-b)(a+b), ] \\ &= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \\ &= \frac{1+\sin A}{\sqrt{\cos^2 A}} \\ &= \frac{1+\sin A}{\cos A} \\ &= \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\ &= \sec A + \tan A \\ &= R.H.S \end{aligned}$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos \theta - \cos \theta} = \tan \theta$$

$$L.H.S = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

Taking Sin  $\theta$  and Cos  $\theta$  common in both numerator and denominator respectively.

$$\Rightarrow \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

By Identity  $\sin^2 A + \cos^2 A = 1$  hence,  $\cos^2 A = 1 - \sin^2 A$  and substituting this in the above equation,

$$\begin{aligned}
&\Rightarrow \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta \{2(1 - \sin^2 \theta) - 1\}} \\
&= \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (2 - 2\sin^2 \theta - 1)} \\
&= \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (1 - 2\sin^2 \theta)} \\
&= \frac{\sin \theta}{\cos \theta} \\
&= \tan \theta \\
&= \text{RHS}
\end{aligned}$$

(viii)  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$\text{By using } (a+b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$\text{By rearranging and using } \sec A = \frac{1}{\cos A} \text{ and } \operatorname{cosec} A = \frac{1}{\sin A}$$

$$\Rightarrow (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2\sin A \left(\frac{1}{\sin A}\right) + 2\cos A \left(\frac{1}{\cos A}\right)$$

$$\text{Hence } (\sin^2 A + \cos^2 A) = 1, \operatorname{cosec}^2 A = (1 + \cot^2 A) \text{ and } (\sec^2 A - \tan^2 A) = 1$$

$$\begin{aligned}
&\Rightarrow 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2 \\
&= 7 + \tan^2 A + \cot^2 A \\
&= \text{R.H.S}
\end{aligned}$$

(ix)  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

$$\text{L.H.S} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \quad \dots \dots \dots (1)$$

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above relations in Equation (1)

$$\begin{aligned}
 & \Rightarrow \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \\
 &= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right) \\
 &= \frac{\cos^2 A \sin^2 A}{\sin A \cos A} \\
 &= \frac{\sin A \cos A}{1} \\
 &= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \quad [\because (\sin^2 A + \cos^2 A) = 1] \\
 &= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \quad [\text{Dividing numerator and denominator by } (\sin A \cos A)] \\
 &= \frac{1}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}} \\
 &= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} \\
 &= \frac{1}{\tan A + \cot A} \\
 &= \text{RHS}
 \end{aligned}$$

$$(x) \left( \frac{1+\tan^2 A}{1+\cot^2 A} \right) = \left( \frac{1-\tan A}{1-\cot A} \right)^2 = \tan^2 A$$

Taking LHS,  $\left( \frac{1+\tan^2 A}{1+\cot^2 A} \right)$

$$= \frac{\sec^2 A}{\cosec^2 A}$$

$$= \frac{1}{\cos^2 A}$$

$$= \frac{1}{\sin^2 A}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \tan^2 A$$

$$= \text{RHS}$$

Taking,  $\left( \frac{1-\tan A}{1-\cot A} \right)^2$

$$= \left( \frac{1-\tan A}{1-\frac{1}{\tan A}} \right)^2$$

$$= \left( \frac{1-\tan A}{\frac{\tan A - 1}{\tan A}} \right)^2$$

$$= \left( (1-\tan A) \times \frac{\tan A}{\tan A - 1} \right)^2$$

$$= (-\tan A)^2$$

$$= \tan^2 A$$

$$= \text{RHS}$$

Hence, L.H.S = R.H.S.