NCERT Solutions for Class 11 Physics Chapter 14 Oscillations

Q. 14.1 Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and bank.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its centre of mass.

(d) An arrow released from a bow

Answer:

(a) The motion is not periodic though it is to and fro.

(b) The motion is periodic.

(c) The motion is periodic.

(d) The motion is not periodic.

Q. 14.2 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

(a) the rotation of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

Answer:

(a) Periodic but not S.H.M.

(b) S.H.M.

(c) S.H.M.

(d) Periodic but not S.H.M.M [A polyatomic molecule has a number of natural frequencies, so its vibration is a superposition of SHM's of a number of different frequencies. This is periodic but not SHM]

Q .14.3 Fig. 14.23 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

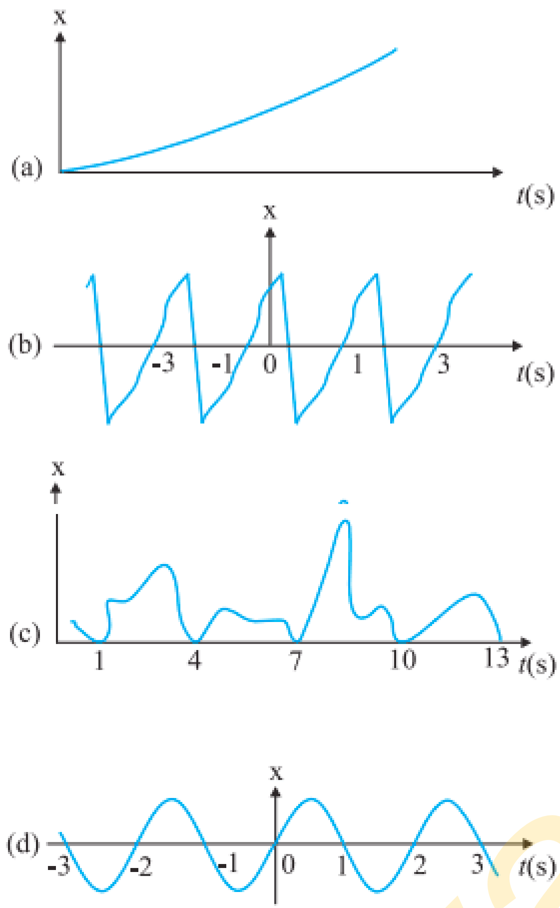


Fig. 14.27

Answer:

The x-t plots for linear motion of a particle in Fig. 14.23 (b) and (d) represent periodic motion with both having a period of motion of two seconds.

Q. 14.4 (a) Which of the following functions of time represent

(a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(a) $sin\ \omega t - cos\ \omega t$

Answer:

Since the above function is of form $Asin(\omega t + \phi)$ it represents SHM with a time period of $\frac{2\pi}{\omega}$

Q. 14.4 (b) Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(b) $sin^3\omega t$

Answer:

The two functions individually represent SHM but their superposition does not give rise to SHM but the motion will definitely be periodic with a period of $\frac{2\pi}{\omega}$

Q. 14.4 (c) Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(c) $3\ cos(\pi/4 - 2\omega t)$

Answer:

The function represents SHM with a period of $\frac{\pi}{\omega}$

Q. 14.4 (c)Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (is any positive constant): (c)

Q.14.4 (d) Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(d) $cos\ \omega t + cos\ 3\omega t + cos\ 5\omega t$

Answer:

Here each individual functions are SHM. But superposition is not SHM. The function represents periodic motion but not SHM.

$$period = LCM(\frac{2\pi}{\omega}, \frac{2\pi}{3\omega}, \frac{2\pi}{5\omega}) = \frac{2\pi}{\omega}$$

Q. 14.4 (e) Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(e) $exp(-\omega^2 t^2)$

Answer:

The given function is exponential and therefore does not represent periodic motion.

Q. 14.4 (f) Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(f) $1 + \omega t + \omega^2 t^2$

Answer:

The given function does not represent periodic motion.

Q. 14.5 (a) A particle is in linear simple harmonic motion between two points, A and B, $10cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

Answer:

Velocity is zero. Force and acceleration are in the positive direction.

Q. 14.5 (b) A particle is in linear simple harmonic motion between two points, A and B , $10cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(b) at the end B ,

Answer:

Velocity is zero. Acceleration and force are negative.

Q. 14.5 (c) A particle is in linear simple harmonic motion between two points, A and B , $10\ cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(c) at the mid-point of AB going towards A ,

Answer:

Velocity is negative that is towards A and its magnitude is maximum. Acceleration and force are zero.

Q. 14.5 (d) A particle is in linear simple harmonic motion between two points, A and B , $10cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(d) at $2cm$ away from B going towards A ,

Answer:

Velocity is negative. Acceleration and force are also negative.

Q.14.5 (e) A particle is in linear simple harmonic motion between two points, A and B , $10cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(e) at $3cm$ away from A going towards B , and

Answer:

Velocity is positive. Acceleration and force are also positive.

Q. 14.5 (f) A particle is in linear simple harmonic motion between two points, A and B , $10cm$ apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(f) at $4cm$ away from B going towards $A.$

Answer:

Velocity, acceleration and force all are negative

Q. 14.6 Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200x^2$

(c) $a = -10x$

(d) $a = 100x^3$

Answer:

Only the relation given in (c) represents simple harmonic motion as the acceleration is proportional in magnitude to the displacement from the midpoint and its direction is opposite to that of the displacement from the mean position.

Q. 14.7 The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\ cos(\omega t + \phi)$.

If the initial $(t = 0)$ position of the particle is $1\ cm$ and its initial velocity is $\omega\ cm/s$, what are its amplitude and initial phase angle ? The angular frequency of the particle is πs^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\ sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Answer:

$\omega = \pi\ rad\ s^{-1}$

$x(t) = Acos(\pi t + \phi)$

at t = 0

$$x(0) = Acos(\pi \times 0 + \phi)$$
$$1 = Acos\phi \qquad (i)$$

$$v = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$$
$$v(t) = -A\pi\, sin(\pi t + \phi)$$

at t = 0

Squaring and adding equation (i) and (ii) we get

Dividing equation (ii) by (i) we get

$$tan\phi = -1$$
$$\phi = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4} \ldots\ldots$$

$$x(t) = Bsin(\pi t + \alpha)$$

at t = 0

$$x(0) = Bsin(\pi \times 0 + \alpha)$$
$$1 = Bsin\alpha \qquad (iii)$$

$$v = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$$
$$v(t) = B\pi cos(\pi t + \alpha)$$

at t = 0

Squaring and adding equation (iii) and (iv) we get

Dividing equation (iii) by (iv) we get

$$tan\alpha = 1$$
$$\alpha = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \ldots\ldots$$

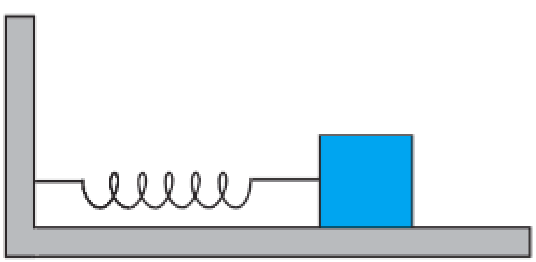
Q. 14.8 A spring balance has a scale that reads from 0 to $50\ kg$. The length of the scale is $20cm$. A body suspended from this balance, when displaced and released, oscillates with a period of $0.6\ s$. What is the weight of the body?

Answer:

Spring constant of the spring is given by

The time period of a spring attached to a body of mass m is given by

Q. 14.9 (i) A spring having with a spring constant $1200\ N\ m^{-1}$ is mounted on a horizontal table as shown in Fig. 14.24. A mass of $3\ kg$ is attached to the free end of the spring. The mass is then pulled sideways to a distance of $2.0\ cm$ and released.



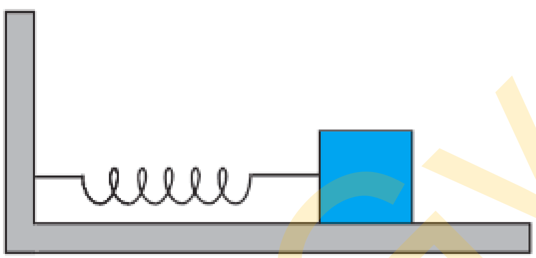
Determine

(i) the frequency of oscillations,

Answer:

The frequency of oscillation of an object of mass m attached to a spring of spring constant k is given by

Q. 14.9 (ii) A spring having with a spring constant $1200\ N\ m^{-1}$ is mounted on a horizontal table as shown in Fig. 14.24. A mass of $3\ kg$ is attached to the free end of the spring. The mass is then pulled sideways to a distance of $2.0\ cm$ and released.



Determine

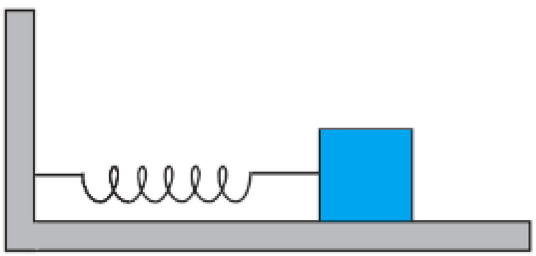
(ii) maximum acceleration of the mass, and

Answer:

A body executing S.H.M experiences maximum acceleration at the extreme points

$$a_{max} = \frac{F_A}{m}$$
$$a_{max} = \frac{kA}{m}$$
$$a_{max} = \frac{1200 \times 0.2}{3}$$
$$a_{max} = 8ms^{-2}$$ (F $_A$ = Force experienced by body at displacement A from mean position)

Q. 14.9 (iii) A spring having with a spring constant $1200\ N\ m^{-1}$ is mounted on a horizontal table as shown in Fig. 14.24. A mass of $3\ kg$ is attached to the free end of the spring. The mass is then pulled sideways to a distance of $2.0\ cm$ and released.



Determine

(iii) the maximum speed of the mass.

Answer:

Maximum speed occurs at the mean position and is given by

$$v_{max} = A\omega$$
$$v_{max} = 0.02 \times 2\pi \times 3.18$$
$$v_{max} = 0.4ms^{-1}$$

Q. 14.10 (a) In Exercise 14.9, let us take the position of mass when the spring is unstreched as $x = 0$,and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch $(t = 0)$, the mass is

(a) at the mean position,

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Answer:

Amplitude is A = 0.02 m

Time period is ω

$$\omega = \sqrt{\frac{k}{m}}$$
$$\omega = \sqrt{\frac{1200}{3}}$$
$$\omega = 20\ rad/s$$

(a) At t = 0 the mass is at mean position i.e. at t = 0, x = 0

$$x(t) = 0.02sin\,(20t)$$

Here x is in metres and t is in seconds.

Q. 14.10 (b) In Exercise 14.9, let us take the position of mass when the spring is unstreched as $x = 0$, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch $(t = 0)$, the mass is

(b) at the maximum stretched position,

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Answer:

Amplitude is A = 0.02 m

Time period is ω

$$\omega = \sqrt{\frac{k}{m}}$$
$$\omega = \sqrt{\frac{1200}{3}}$$
$$\omega = 20\ rad/s$$

(b) At t = 0 the mass is at the maximum stretched position.

x(0) = A

$$\phi = \frac{\pi}{2}$$

$$x(t) = 0.02sin\left(20t + \frac{\pi}{2}\right)$$
$$x(t) = 0.02cos(20t)$$

Here x is in metres and t is in seconds.

Q. 14.10 (c) In Exercise 14.9, let us take the position of mass when the spring is unstreched as $x = 0$,and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch $(t = 0)$, the mass is

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Answer:

Amplitude is A = 0.02 m

Time period is ω

$$\omega = \sqrt{\frac{k}{m}}$$
$$\omega = \sqrt{\frac{1200}{3}}$$
$$\omega = 20\ rad/s$$

(c) At t = 0 the mass is at the maximum compressed position.

x(0) = -A

$$\phi = \frac{3\pi}{2}$$

$$x(t) = 0.02sin\left(20t + \frac{3\pi}{2}\right)$$
$$x(t) = -0.02cos(20t)$$

Here x is in metres and t is in seconds.

The above functions differ only in the initial phase and not in amplitude or frequency.

Q. 14.11 Figures 14.25 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.

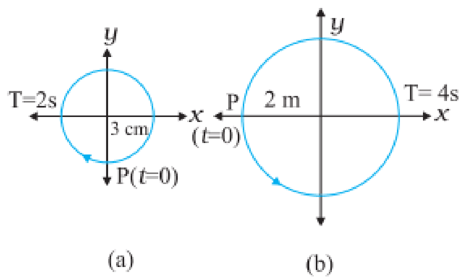


Fig. 14.25

Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Answer:

(a) Let the required function be $x(t) = asin(\pm\omega t + \phi)$

Amplitude = 3 cm = 0.03 m

T = 2 s

$$\omega = \frac{2\pi}{T}$$
$$\omega = \pi rad\ s$$

Since initial position x(t) = 0, $\phi = 0$

As the sense of revolution is clock wise

$$x(t) = 0.03sin(-\omega t)$$
$$x(t) = -0.03sin(\pi t)$$

Here x is in metres and t is in seconds.

(b)Let the required function be $x(t) = asin(\pm\omega t + \phi)$

Amplitude = 2 m

T = 4 s

$$\omega = \frac{2\pi}{T}$$
$$\omega = \frac{\pi}{2} rad\ s$$

Since initial position x(t) = -A, $\phi = \frac{3\pi}{2}$

As the sense of revolution is anti-clock wise

$$x(t) = 2sin(\omega t + \frac{3\pi}{2})$$
$$x(t) = -2cos(\frac{\pi}{2}t)$$

Here x is in metres and t is in seconds.

Q. 14.12 (a) Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial $(t = 0)$ position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2\ sin(3t + \pi/3)$

Answer:

$$x = -2\sin(3t + \pi/3)$$
$$x = 2cos(3t + \frac{\pi}{3} + \frac{\pi}{2})$$
$$x = 2cos(3t + \frac{5\pi}{6})$$

The initial position of the particle is x(0)

$$x(0) = 2cos(0 + \frac{5\pi}{6})$$
$$x(0) = 2cos(\frac{5\pi}{6})$$
$$x(0) = -\sqrt{3}cm$$

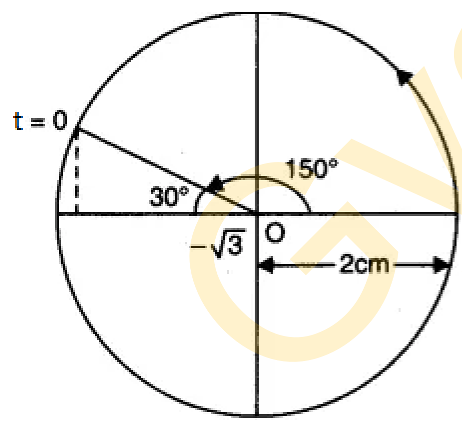
The radius of the circle i.e. the amplitude is 2 cm

The angular speed of the rotating particle is $\omega = 3rad\ s^{-1}$

Initial phase is

$$\phi = \frac{5\pi}{6}$$
$$\phi = 150^o$$

The reference circle for the given simple Harmonic motion is



Q. 14.12 (b) Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial $(t = 0)$ position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(b) $x = cos(\pi/6 - t)$

Answer:

$$x(t) = cos(\frac{\pi}{6} - t)$$
$$x(t) = cos(t - \frac{\pi}{6})$$

The initial position of the particle is x(0)

$$x(0) = cos(0 - \frac{\pi}{6})$$
$$x(0) = cos(\frac{\pi}{6})$$
$$x(0) = \frac{\sqrt{3}}{2}cm$$

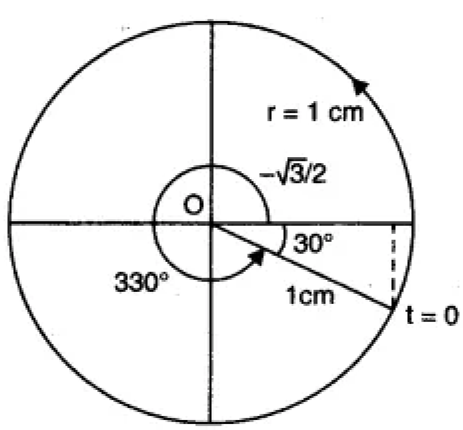
The radius of the circle i.e. the amplitude is 1 cm

The angular speed of the rotating particle is $\omega = 1rad\ s^{-1}$

Initial phase is

$$\phi = -\frac{\pi}{6}$$
$$\phi = -30^o$$

The reference circle for the given simple Harmonic motion is



Q. 14.12 (c) Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial $(t = 0)$ position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(c) $x = 3\ sin(2\pi t + \pi/4)$

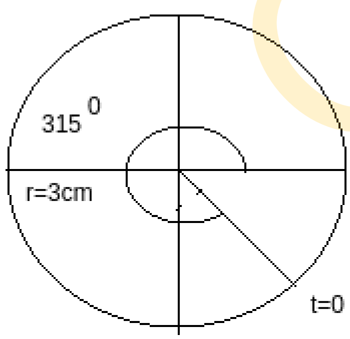
Answer:

$x = 3\ sin(2\pi t + \pi/4)$

At t= 0

$phase = \frac{7\pi}{4}$

Reference circle is as follows



Oscillations Excercise:

Question:

Q. 14.12 (d) Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial $(t = 0)$ position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(d) $x = 2\ cos\ \pi t$

Answer:

$x(t) = 2cos(\pi t)$

The initial position of the particle is x(0)

$x(0) = 2cos(0)$
$x(0) = 2cm$

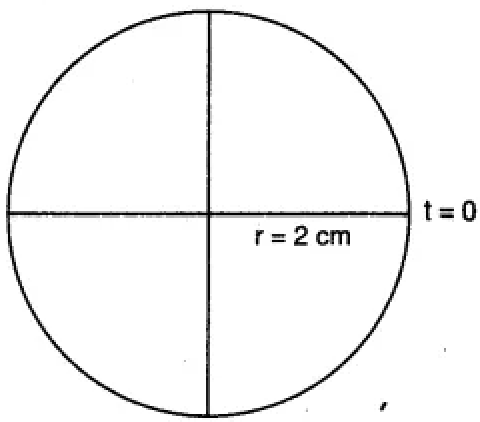
The radius of the circle i.e. the amplitude is 2 cm

The angular speed of the rotating particle is $\omega = \pi rad\ s^{-1}$

Initial phase is

$\phi = 0^o$

The reference circle for the given simple Harmonic motion is



Q. 14.13 (a) Figure 14.30

(a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30

(b) is stretched by the same force F.

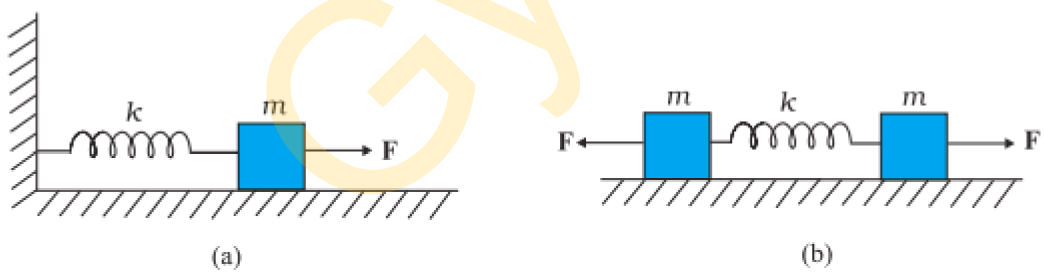


Fig. 14.30

(a) What is the maximum extension of the spring in the two cases?

Answer:

(a) Let us assume the maximum extension produced in the spring is x.

At maximum extension

$$F = Kx$$
$$x = \frac{F}{k}$$

(b) Let us assume the maximum extension produced in the spring is x. That is x/2 due to force towards left and x/2 due to force towards right

$$F = k\frac{x}{2} + k\frac{x}{2}$$
$$\Rightarrow x = \frac{F}{k}$$

Oscillations Excercise:

Question:

Q. 14.13 (b) Figure 14.26 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.26 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.26 (b) is stretched by the same force F .

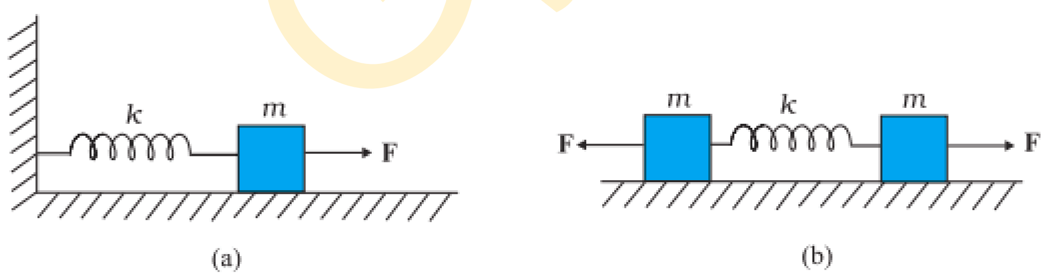


Fig. 14.30

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

Answer:

(b).(a) In Fig, (a) we have

F=-kx

ma=-kx

$$a = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(b) In fig (b) the two equal masses will be executing SHM about their centre of mass. The time period of the system would be equal to a single object of same mass m attached to a spring of half the length of the given spring (or undergoing half the extension of the given spring while applied with the same force)

Spring constant of such a spring would be 2k

F=-2kx

ma=-2kx

Q. 14.14 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of $1.0m$ If the piston moves with simple harmonic motion with an angular frequency of $200\ rad/min$, what is its maximum speed ?

Answer:

Amplitude of SHM = 0.5 m

angular frequency is

$$\omega = 200\ rad/min$$
$$\omega = 3.33\ rad/s$$

If the equation of SHM is given by

$$x(t) = Asin(\omega t + \phi)$$

The velocity would be given by

The maximum speed is therefore

$$v_{max} = A\omega$$
$$v_{max} = 0.5 \times 3.33$$
$$v_{max} = 1.67ms^{-1}$$

Q. 14.15 The acceleration due to gravity on the surface of moon is $1.7m\ s^{-2}$ What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is $3.5\ s$? (g on the surface of earth is 9.8 m s–2)

Answer:

The time period of a simple pendulum of length l executing S.H.M is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$g_e = 9.8\ m\ s^{-2}$

$g_m = 1.7\ m\ s^{-2}$

The time period of the pendulum on the surface of Earth is $T_e = 3.5$ s

The time period of the pendulum on the surface of the moon is T_m

Q. 14.16 (a) Answer the following questions :

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle:

$T = 2\pi\sqrt{\frac{m}{k}}.$ A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

Answer:

In case of spring, the spring constant is independent of the mass attached whereas in case of a pendulum k is proportional to m making k/m constant and thus the time period comes out to be independent of the mass of the body attached.

Q. 14.16 (b) Answer the following questions :

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations: For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}.$ Think of a qualitative argument to appreciate this result.

Answer:

In reaching the result $T = 2\pi\sqrt{\frac{l}{g}}$ we have assumed sin(x/l)=x/l. This assumption is only true for very small values of x $(or\ \theta)$. Therefore it is obvious that once x takes larger values we will have deviations from the above-mentioned value.

Q. 14.16 (c) Answer the following questions :

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

Answer:

The watch must be using an electrical circuit or a spring system to tell the time and therefore free falling would not affect the time his watch predicts.

Q. 14.16 (d) Answer the following questions :

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Answer:

While free falling the effective value of g inside the cabin will be zero and therefore the frequency of oscillation of a simple pendulum would be zero i.e. it would not vibrate at all because of the absence of a restoring force.

Q .14.17 A simple pendulum of length l and having a bob of mass M is suspended in a car.The car is moving on a circular track of radius R with a uniform speed V. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Answer:

Acceleration due to gravity = g (in downwards direction)

Centripetal acceleration due to the circular movement of the car = a c

$a_c = \frac{v^2}{R}$ (in the horizontal direction)

Effective acceleration is

$$g' = \sqrt{g^2 + a_c^2}$$
$$g' = \sqrt{g^2 + \frac{v^4}{R^2}}$$

The time period is T'

$$T' = 2\pi\sqrt{\frac{l}{g'}}$$
$$T' = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

Q. 14.18 A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released

Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Answer:

Let the cork be displaced by a small distance x in downwards direction from its equilibrium position where it is floating.

The extra volume of fluid displaced by the cork is Ax

Taking the downwards direction as positive we have

Comparing with a=-kx we have

$$k = \frac{\rho_1 g}{\rho h}$$
$$T = \frac{2\pi}{\sqrt{k}}$$
$$T = 2\pi\sqrt{\frac{\rho h}{\rho_1 g}}$$

Q. 14.19 One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Answer:

Let the height of each mercury column be h.

The total length of mercury in both the columns = 2h.

Let the cross-sectional area of the mercury column be A.

Let the density of mercury be ρ

When either of the mercury columns dips by a distance x, the total difference between the two columns becomes 2x.

Weight of this difference is $2Ax\rho g$

This weight drives the rest of the entire column to the original mean position.

Let the acceleration of the column be a Since the force is restoring

$$2hA\rho(-a) = 2xA\rho g$$
$$a = -\frac{g}{h}x$$

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{g}{h}x$ **which is the equation of a body executing S.H.M**

The time period of the oscillation would be

$$T = 2\pi\sqrt{\frac{h}{g}}$$

NCERT solutions for class 11 physics chapter 14 oscillations additional exercise

Q. 14.20 An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released , it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see Fig. 14.33].

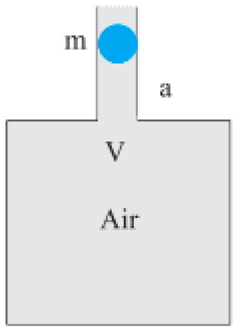


Fig.14.33

Answer:

Let the initial volume and pressure of the chamber be V and P.

Let the ball be pressed by a distance x.

This will change the volume by an amount ax.

Let the change in pressure be ΔP

Let the Bulk's modulus of air be K.

$$K = \frac{\Delta P}{\Delta V / V}$$
$$\Delta P = \frac{Kax}{V}$$

This pressure variation would try to restore the position of the ball.

Since force is restoring in nature displacement and acceleration due to the force would be in different directions.

The above is the equation of a body executing S.H.M.

The time period of the oscillation would be

$$T = \frac{2\pi}{a}\sqrt{\frac{mV}{k}}$$

Q. 14.21 (a) You are riding in an automobile of mass $3000\ kg.$. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags $15\ cm$ when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by $50^o/_o$ during one complete oscillation. Estimate the values of

(a) the spring constant K

Answer:

Mass of automobile (m) = 3000 kg

There are a total of four springs.

Compression in each spring, x = 15 cm = 0.15 m

Let the spring constant of each spring be k

$$4kx = mg$$
$$k = \frac{3000 \times 9.8}{4 \times 0.15}$$
$$k = 4.9 \times 10^4\ N$$

Q. 14.21 (b) You are riding in an automobile of mass $3000kg$. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags $15\ cm$ when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by $50^o/_o$ during one complete oscillation. Estimate the values of

(b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports $750\ kg.$.

Answer:

The amplitude of oscillation decreases by 50 % in one oscillation i.e. in one time period.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi \times \sqrt{\frac{3000}{4 \times 4.9 \times 10^4}}$$
$$T = 0.77\ s$$

For damping factor b we have

$$x = x_0 e^{\left(-\frac{bt}{2m}\right)}$$

$x = x_0/2$

$t = 0.77s$

$m = 750$ kg

14.22 Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Answer:

Let the equation of oscillation be given by $x = Asin(\omega t)$

Velocity would be given as

$$v = \frac{dx}{dt}$$
$$v = A\omega cost(\omega t)$$

Kinetic energy at an instant is given by

Time Period is given by

$$T = \frac{2\pi}{\omega}$$

The Average Kinetic Energy would be given as follows

The potential energy at an instant T is given by

The Average Potential Energy would be given by

We can see K $_{av}$ = U $_{av}$

Q .14.23 A circular disc of mass $10\ kg$ is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be $1.5\ s.$ The radius of the disc is $15\ cm$. Determine the torsional spring constant of the wire.

(Torsional spring constant a is defined by the relation $J = -a\ \theta$, where J is the restoring couple and θ the angle of twist).

Answer:

$J = -a\ \theta$

Moment of Inertia of the disc about the axis passing through its centre and perpendicular to it is

$$I = \frac{MR^2}{2}$$

The period of Torsional oscillations would be

Q. 14.24 (a) A body describes simple harmonic motion with an amplitude of $5\ cm$ and a period of $0.2\ s$. Find the acceleration and velocity of the body when the displacement is

(a) $5\ cm$

Answer:

A = 5 cm = 0.05 m

T = 0.2 s

$$\omega = \frac{2\pi}{T}$$
$$\omega = \frac{2\pi}{0.2}$$
$$\omega = 10\pi\ rad\ s^{-1}$$

At displacement x acceleration is $a = -\omega^2 x$

At displacement x velocity is $v = \omega\sqrt{A^2 - x^2}$

(a)At displacement 5 cm

$$v = 10\pi\sqrt{(0.05)^2 - (0.05)^2}$$
$$v = 0$$

$$a = -(10\pi)^2 \times 0.05$$
$$a = -49.35ms^{-2}$$

Q. 14.24 (b) A body describes simple harmonic motion with an amplitude of $5\ cm$ and a period of $0.2\ s$. Find the acceleration and velocity of the body when the displacement is

(b) $3\ cm$

Answer:

A = 5 cm = 0.05 m

T = 0.2 s

$$\omega = \frac{2\pi}{T}$$
$$\omega = \frac{2\pi}{0.2}$$
$$\omega = 10\pi\ rad\ s^{-1}$$

At displacement x acceleration is $a = -\omega^2 x$

At displacement x velocity is $v = \omega\sqrt{A^2 - x^2}$

(a)At displacement 3 cm

$$v = 10\pi\sqrt{(0.05)^2 - (0.03)^2}$$
$$v = 10\pi\sqrt{0.0016}$$
$$v = 10\pi \times 0.04$$
$$v = 1.257ms^{-1}$$

$$a = -(10\pi)^2 \times 0.03$$
$$a = -29.61ms^{-2}$$

Q. 14.24 (c) A body describes simple harmonic motion with an amplitude of $5\ cm$ and a period of $0.2\ s.$ Find the acceleration and velocity of the body when the displacement is

(c) $0\ cm$

Answer:

A = 5 cm = 0.05 m

T = 0.2 s

$$\omega = \frac{2\pi}{T}$$
$$\omega = \frac{2\pi}{0.2}$$
$$\omega = 10\pi\ rad\ s^{-1}$$

At displacement x acceleration is $a = -\omega^2 x$

At displacement x velocity is $v = \omega\sqrt{A^2 - x^2}$

(a)At displacement 0 cm

$$v = 10\pi\sqrt{(0.05)^2 - (0)^2}$$
$$v = 10\pi \times 0.05$$
$$v = 1.57ms^{-1}$$

$$a = -(10\pi)^2 \times 0$$
$$a = 0$$

Q. 14.25 A mass attached to a spring is free to oscillate, with angular velocity ω , in a horizontal plane without friction or damping. It is pulled to distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$ Determine the amplitude of the resulting oscillations in terms of the parameters ω , x_0 and v_0 . [Hint : Start with the equation $x = a\ cos(\omega t + \theta)$ and note that the initial velocity is negative.]

Answer:

At the maximum extension of spring, the entire energy of the system would be stored as the potential energy of the spring.

Let the amplitude be A

The angular frequency of a spring-mass system is always equal to $\sqrt{\frac{k}{m}}$

Therefore

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$