NCERT Solutions Class 12 Maths Chapter 2
Miscellaneous Exercise

Question 1:

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Solution:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)=\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$$
$$=\cos^{-1}\left[\cos\frac{\pi}{6}\right]$$
$$=\frac{\pi}{6}$$

Question 2:

Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$.

Solution:

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left[\tan\left(2\pi-\frac{5\pi}{6}\right)\right]$$
$$=\tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right]$$
$$=\tan^{-1}\left[\tan\left(\pi-\frac{5\pi}{6}\right)\right]$$
$$=\tan^{-1}\left[\tan\frac{\pi}{6}\right]$$
$$=\frac{\pi}{6}$$

Question 3:

Prove that $2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$.

Solution:

Let $\sin^{-1}\frac{3}{5}=x \Rightarrow \sin x=\frac{3}{5}$

Then,

$$\cos x = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

Therefore,

$$\tan x = \frac{3}{4}$$

$$x = \tan^{-1}\frac{3}{4}$$

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \qquad \ldots(1)$$

Thus,

$$LHS = 2\sin^{-1}\frac{3}{5}$$

$$= 2\tan^{-1}\frac{3}{4} \qquad \left[from\ (1)\right]$$

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right)$$

$$= RHS$$

Question 4:

Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$.

Solution:

Let $\sin^{-1}\frac{8}{17} = x \Rightarrow \sin x = \frac{8}{17}$

Then,

$$\cos x = \sqrt{1-\left(\frac{8}{17}\right)^2} = \sqrt{\frac{225}{289}} = \frac{15}{17}$$

Therefore,

$$\tan x = \frac{8}{15}$$

$$x = \tan^{-1}\frac{8}{15}$$

$$\sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15} \qquad \ldots(1)$$

Now, let $\sin^{-1}\frac{3}{5}=y \Rightarrow \sin y=\frac{3}{5}$

Then,

$$\cos y=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

Therefore,

$$\tan y=\frac{3}{4}$$

$$y=\tan^{-1}\frac{3}{4}$$

$$\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4} \qquad \ldots(2)$$

Thus, by using (1) and (2)

$$\begin{aligned} LHS &= \sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5} \\ &= \tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4} \\ &= \tan^{-1}\left[\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\cdot\frac{3}{4}}\right] \\ &= \tan^{-1}\left[\frac{\frac{32+45}{60}}{\frac{60-24}{60}}\right] \\ &= \tan^{-1}\frac{77}{36} \\ &= RHS \end{aligned}$$

Question 5:

Prove that $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$.

Solution:

Let $\cos^{-1}\frac{4}{5}=x \Rightarrow \cos x=\frac{4}{5}$

Then,

$$\sin x=\sqrt{1-\left(\frac{4}{5}\right)^2}=\frac{3}{5}$$

Therefore,

$$\tan x=\frac{3}{4}$$

$$x=\tan^{-1}\frac{3}{4}$$

$$\cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4} \qquad \ldots(1)$$

Now, let $\cos^{-1}\frac{12}{13}=y\Rightarrow\cos y=\frac{12}{13}$

Then,

$$\sin y=\frac{5}{13}$$

Therefore,

$$\tan y=\frac{5}{12}$$

$$y=\tan^{-1}\frac{5}{12}$$

$$\cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12} \qquad \ldots(2)$$

Thus, by using (1) and (2)

$$\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12}$$

$$=\tan^{-1}\left[\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}\right]$$

$$=\tan^{-1}\left[\frac{56}{33}\right] \qquad \ldots(3)$$

Now, let $\cos^{-1}\frac{33}{65}=z\Rightarrow\cos z=\frac{33}{65}$

Then,

$$\sin z=\sqrt{1-\left(\frac{33}{65}\right)^2}=\frac{56}{65}$$

Therefore,

$$\tan z = \frac{33}{56}$$

$$z = \tan^{-1}\frac{56}{33}$$

$$\cos^{-1}\frac{33}{65} = \tan^{-1}\frac{56}{33} \qquad \ldots(4)$$

Thus, by using (3) and (4)

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$$

Hence proved.

Question 6:

Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$.

Solution:

Let $\cos^{-1}\frac{12}{13} = y \Rightarrow \cos y = \frac{12}{13}$

Then,

$$\sin y = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}$$

Therefore,

$$\tan y = \frac{5}{12}$$

$$y = \tan^{-1}\frac{5}{12}$$

$$\cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12} \qquad \ldots(1)$$

Now, let $\sin^{-1}\frac{3}{5} = x \Rightarrow \sin x = \frac{3}{5}$

Then,

$$\cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

Therefore,

$$\tan x = \frac{3}{4}$$

$$x = \tan^{-1}\frac{3}{4}$$

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \qquad \ldots(2)$$

Now, let $\sin^{-1}\frac{56}{65} = z \Rightarrow \sin z = \frac{56}{65}$

Then,

$$\cos z = \sqrt{1-\left(\frac{56}{65}\right)^2} = \frac{33}{65}$$

Therefore,

$$\tan z = \frac{56}{33}$$

$$z = \tan^{-1}\frac{56}{33}$$

$$\sin^{-1}\frac{56}{65} = \tan^{-1}\frac{56}{33} \qquad \ldots(3)$$

Thus, by using (1) and (2)

$$LHS = \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{20+36}{48}}{\frac{48-15}{48}}\right]$$

$$= \tan^{-1}\left(\frac{56}{33}\right)$$

$$= \sin^{-1}\frac{56}{65} \qquad \left[\text{Using}(3)\right]$$

$$= RHS$$

Question 7:

Prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$.

Solution:

Let $\sin^{-1}\frac{5}{13}=x \Rightarrow \sin x=\frac{5}{13}$

Then,

$$\cos x=\sqrt{1-\left(\frac{5}{13}\right)^2}=\frac{12}{13}$$

Therefore,

$$\tan x=\frac{5}{12}$$

$$x=\tan^{-1}\frac{5}{12}$$

$$\sin^{-1}\frac{5}{13}=\tan^{-1}\frac{5}{12} \qquad \ldots(1)$$

Now, let $\cos^{-1}\frac{3}{5}=y \Rightarrow \cos y=\frac{3}{5}$

Then,

$$\sin y=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$$

Therefore,

$$\tan y=\frac{4}{3}$$

$$y=\tan^{-1}\frac{4}{3}$$

$$\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{4}{3} \qquad \ldots(2)$$

Thus, by using (1) and (2)

$$RHS = \sin^{-1}\frac{5}{12} + \cos^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$$

$$= \tan^{-1}\left(\frac{63}{16}\right)$$

$$= LHS$$

Question 8:

Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Solution:

$$LHS = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\cdot\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\cdot\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$= RHS$$

Question 9:

Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$.

Solution:

Let $x=\tan^2\theta$

Then,

$$\sqrt{x}=\tan\theta$$
$$\theta=\tan^{-1}\sqrt{x}$$

Therefore,

$$\left(\frac{1-x}{1+x}\right)=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$=\cos 2\theta$$

Thus,

$$RHS=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$$
$$=\frac{1}{2}\cos^{-1}(\cos 2\theta)$$
$$=\frac{1}{2}\times 2\theta$$
$$=\theta$$
$$=\tan^{-1}\sqrt{x}$$
$$=LHS$$

Question 10:

Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$.

Solution:

$$\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2} \quad (by\ rationalizing)$$
$$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$$
$$=\frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x}=\frac{1+\cos x}{\sin x}$$
$$=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$
$$=\cot\frac{x}{2}$$

Thus,

$$LHS = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$
$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$
$$= \frac{x}{2}$$
$$= RHS$$

Question 11:

Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{2}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}}\le x\le 1$

Solution:

Let $x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$

Thus,

$$LHS = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$
$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$
$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$
$$= \tan^{-1}1-\tan^{-1}(\tan\theta)$$
$$= \frac{\pi}{4}-\theta$$
$$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$
$$= RHS$$

Question 12:

Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Solution:

$$\begin{aligned}LHS &= \frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\\ &= \frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)\\ &= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right) \qquad \ldots(1)\end{aligned}$$

Now, let $\cos^{-1}\frac{1}{3}=x \Rightarrow \cos x=\frac{1}{3}$

Therefore,

$$\begin{aligned}\sin x &= \sqrt{1-\left(\frac{1}{3}\right)^2}\\ &= \frac{2\sqrt{2}}{3}\\ x &= \sin^{-1}\frac{2\sqrt{2}}{3}\\ \cos^{-1}\frac{1}{3} &= \sin^{-1}\frac{2\sqrt{2}}{3} \qquad \ldots(2)\end{aligned}$$

Thus, by using (1) and (2)

$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

Hence proved.

Question 13:

Solve $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$.

Solution:

It is given that $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

Since, $2\tan^{-1}(x)=\tan^{-1}\frac{2x}{1-x^2}$

Hence,

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \left(\frac{2\cos x}{1-\cos^2 x}\right) = (2\operatorname{cosec} x)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$$

$$\Rightarrow \cos x = \sin x$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow \tan x = \tan\frac{\pi}{4}$$

Therefore,

$x = n\pi + \frac{\pi}{4}$, where $n \in Z$.

Question 14:

Solve $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x, (x > 0)$

Solution:

Since $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$

Hence,

$$\Rightarrow \tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1} x$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

Question 15:

Solve $\sin\left(\tan^{-1} x\right), |x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Solution:

Let $\tan y = x$

Therefore,

$$\sin y = \frac{x}{\sqrt{1+x^2}}$$

Now, let $\tan^{-1} x = y$

Therefore,

$$y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Hence,

$$\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Thus,

$$\sin\left(\tan^{-1} x\right) = \sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$$

$$= \frac{x}{\sqrt{1+x^2}}$$

Thus, the correct option is D.

Question 16:

Solve: $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x is equal to

(A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

Solution:

It is given that $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1} x = \frac{\pi}{2} - \sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1} x = \cos^{-1}(1-x) \qquad \ldots(1)$$

Let $\sin^{-1} x = y \Rightarrow \sin y = x$

Hence,

$$\cos y = \sqrt{1-x^2}$$
$$y = \cos^{-1}\left(\sqrt{1-x^2}\right)$$
$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$$

From equation (1), we have

$$-2\cos^{-1}\sqrt{1-x^2} = \cos^{-1}(1-x)$$

Put $x = \sin y$

$$\Rightarrow -2\cos^{-1}\sqrt{1-\sin^2 y} = \cos^{-1}(1-\sin y)$$
$$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$$
$$\Rightarrow -2y = \cos^{-1}(1-\sin y)$$
$$\Rightarrow 1-\sin y = \cos(-2y)$$
$$\Rightarrow 1-\sin y = \cos 2y$$
$$\Rightarrow 1-\sin y = 1-2\sin^2 y$$
$$\Rightarrow 2\sin^2 y - \sin y = 0$$
$$\Rightarrow \sin y(2\sin y - 1) = 0$$
$$\Rightarrow \sin y = 0, \frac{1}{2}$$

Therefore,

$$x = 0, \frac{1}{2}$$

When $x = \frac{1}{2}$, it does not satisfy the equation.
Hence, $x = 0$ is the only solution

Thus, the correct option is C.

Question 17:

Solve $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{-3\pi}{4}$

Solution:

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}=\tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right]$$

$$=\tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$=\tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$$

$$=\tan^{-1}(1)$$

$$=\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$=\frac{\pi}{4}$$

Thus, the correct option is C.